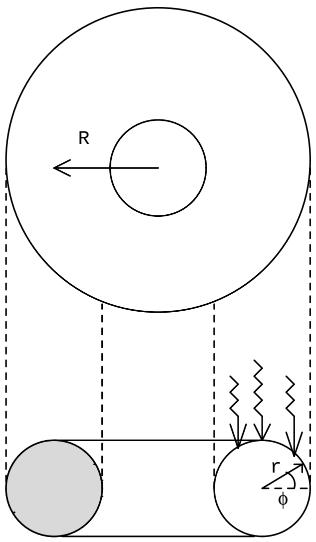
Delft University of Technology

Faculty of Electrical Eng, Mathematics and Computer Science

**Computer Assignment 1.1.3**
**Numerical Analysis CII (wi4014)**
**Numerical Methods PDE (wi3001)**




The sun shines on the toroidal object in the figure causing a heat flow into the object. The objective of this assignment is to calculate the thermal equilibrium in the object. Except for the solar radiation there are no heat sources.
The mathematical formulation of this problem is:

$$-\Delta T = 0 \tag{1}$$

$$\frac{\partial T}{\partial n} + \alpha T = f \quad \text{on the boundary} \tag{2}$$

In this equation $T = (T_{torus} - T_{env}) *$ radiation constant, in which $T_{env}$ is the environment temperature.
$f$ represents the heat source and $\alpha$ is a material constant.
The main radius of the torus is $R$, the secondary radius $r$.
The source term $f$ is given by:

$$f = \begin{cases} \sin\phi & \text{if} \quad 0 < \phi < \pi \\ 0 & \text{if} \quad \pi < \phi < 2\pi \end{cases}$$

Assignment:

a. Derive the minimization problem for (1) with boundary condition (2).

b. Which symmetries can be used in solving this problem?

c. To approximate the solution to this minimization problem we use the Finite Element Method. The elements in the cross section have triangular shape and the approximation is linear. Derive the element matrices and vectors for the internal elements.
Derive element matrices and vectors for the boundary elements..

d. Solve the problem with the SEPRAN package.
Use $r = 2, R = 4$.
Choose $\alpha = 0.75$ and solve the problem for two different grid sizes.
Print the temperatures and make a contour plot.

e. Explain qualitatively the main characteristics of the contour plot.

f. Perform the same calculations with $\alpha = 80$.
   Explain why temperatures are lower in this case compared to d.

**Remark**

The assignment has to be submitted in the form of a report providing mathematical background, derivation and results. The above questions may be used as guidance to make the report, but a report should contain more than merely a list of answers to the questions.