DELFT UNIVERSITY OF TECHNOLOGY
FACULTY OF INFORMATION TECHNOLOGY AND SYSTEMS

Numerieke Wiskunde b.o. (wi4012)
Numerieke Analyse CII (wi4014)
Numerieke Methoden PDV (wi3001)
2000/2001 First Series.

1. Excercise 3.10 (page 48)

2. Instead of (3.17) you also could solve:

$$-\frac{d}{dx}e^{-vx}\frac{dy}{dx} = 0 \ , \quad y(0) = 0 \ , \quad y(1) = 1$$

   Using central differences (see excercise 3.3.a page 36) one obtains a positive definite matrix A.

   (a) Show that the solution of this problem does not exhibit wiggles. Hint: supposing both $u_{i-1} > u_i$ and $u_{i+1} > u_i$ hold, try to arrive at a contradiction.

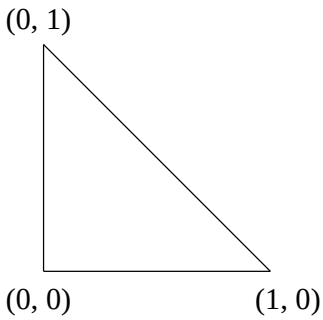
   (b) Extend this method to non-constant velocities, that is:

$$-y^{''} + v(x)y^{'} = f \ , \quad y(0) = 0 \ , \quad y(1) = 1 \ .$$

   Hint: consider $\phi(x) = \int_0^x v(t)\, dt$.

   Unfortunately this method does not work in two or three dimensions.

3. In the triangle (0, 0), (1, 0), (0, 1) (see figure) the PDE



$$-\text{div}\, k\nabla\varphi + \boldsymbol{u}\cdot\nabla\varphi = f$$

   holds, in which φ, k, f and $\boldsymbol{u}$ are functions of x and y. We suppose, that the flow is incompressible:

$$\text{div}\, \boldsymbol{u} = 0$$

At the boundaries we have the following boundary conditions:

$$\begin{array}{lll} x = 0 & : & \varphi = g_1 \\ y = 0 & : & \sigma\varphi + k\frac{\partial\varphi}{\partial n} = g_2 \end{array} \qquad\qquad x - y = 1 \;:\; k\frac{\partial\varphi}{\partial n} = 0$$

This problem is solved by a finite volume method with an equidistant rectangular grid.

(a) Derive an $O(h^2)$ discretization molecule for the inner region. Hint: from the incompressibility condition, the equality

$$\boldsymbol{u} \cdot \nabla\varphi = \operatorname{div}(\boldsymbol{u}\varphi)$$

follows.

(b) Show how the boundary conditions at each of the three boundaries have to be treated in order to maintain an accuracy of $O(h^2)$. Choose the grid in such a way, that all boundaries contain nodes.

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