Multi Grid Presentation

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1: Presets: $\mathbf{u}_h^0$, $\mathbf{r}_h^0 = \mathbf{f}_h - A\mathbf{u}_h^0$

2: $\mathbf{u}_h^{\mathsf{prs}} = S(\mathbf{u}_h^0, \mathbf{b}, A, n_0)\{\text{Presmoothing}\}$

3: $\mathbf{r}_H = R_{Hh}\mathbf{r}_h$

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2: $\mathbf{u}_h^{\mathsf{prs}} = S(\mathbf{u}_h^0, \mathbf{b}, A, n_0)\{\text{Presmoothing}\}$

3: $\mathbf{r}_H = R_{Hh}\mathbf{r}_h$

4: Solve $A_H \mathbf{c}_H = \mathbf{r}_H$

5: $\mathbf{u}_h^{\mathsf{cgc}} = \mathbf{u}_h^p + P_{hH}\mathbf{c}_H$ {Coarse Grid Correction}

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Two Grid Algorithm

1: Presets: $\mathbf{u}_h^0$, $\mathbf{r}_h^0 = \mathbf{f}_h - A\mathbf{u}_h^0$

2: $\mathbf{u}_h^{\mathsf{prs}} = S(\mathbf{u}_h^0, \mathbf{b}, A, n_0)$\{Presmoothing\}

3: $\mathbf{r}_H = R_{Hh}\mathbf{r}_h$

4: Solve $A_H\mathbf{c}_H = \mathbf{r}_H$

5: $\mathbf{u}_h^{\mathsf{cgc}} = \mathbf{u}_h^p + P_{hH}\mathbf{c}_H$ \{Coarse Grid Correction\}

6: $\mathbf{u}_h^{\mathsf{pos}} = S(\mathbf{u}_h^{\mathsf{cgc}}, \mathbf{b}, A, n_1)$\{Postsmoothing\}

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MGRecursive $(A_\ell, \mathbf{r}_\ell, \mathbf{c}_\ell, \ell)$

if $\ell < p - 1$ **then**

$\quad \mathbf{c}_\ell = S(\mathbf{0}, \mathbf{r}_\ell, A_\ell, n_0)$ {Presmoothing}

else

$\quad$ Solve $A_{p-1}\mathbf{c}_{p-1} = \mathbf{r}_{p-1}$ {Direct solution on coarsest level}

end if

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if $\ell < p - 1$ **then**

$\quad \mathbf{c}_\ell = S(\mathbf{0}, \mathbf{r}_\ell, A_\ell, n_0)$ {Presmoothing}

$\quad \mathbf{r}_{\ell+1} = R_\ell(\mathbf{r}_\ell - A_\ell \mathbf{c}_\ell)$ {Calculate coarse grid residual}

$\quad A_{\ell+1} = R_\ell A_\ell P_{\ell+1}$ {Calculate coarse grid matrix}

$\quad$**else**

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  $A_{\ell+1} = R_\ell A_\ell P_{\ell+1}$ {Calculate coarse grid matrix}

  call MGRecursive $(A_{\ell+1}, \mathbf{r}_{\ell+1}, \mathbf{c}_{\ell+1}, \ell + 1)$

  $\mathbf{c}_\ell = \mathbf{c}_\ell + P_{\ell+1} \mathbf{c}_{\ell+1}$ {Coarse grid correction}

else

  Solve $A_{p-1}\mathbf{c}_{p-1} = \mathbf{r}_{p-1}$ {Direct solution on coarsest level}

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Multi Grid Algorithm

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if $\ell < p - 1$ **then**

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    $\mathbf{c}_\ell = \mathbf{c}_\ell + P_{\ell+1} \mathbf{c}_{\ell+1}$ {Coarse grid correction}

    $\mathbf{c}_\ell = S(\mathbf{c}_\ell, \mathbf{r}_\ell, A_\ell, n_1)$ {Postsmoothing}

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    Solve $A_{p-1} \mathbf{c}_{p-1} = \mathbf{r}_{p-1}$ {Direct solution on coarsest level}

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