

# Welcome to EE4550

## Electromagnetic Modeling in Power Engineering

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EEMCS Faculty - TU Delft

# Outline

- 1 Course Overview, Study Goals, Assessment, Preliminaries and Deadlines**
- 2 Block 1/3: Electrostatics - Finite Differences
- 3 Block 2/3: Magnetostatics - Finite Elements Introduction
- 4 Block 3/3: Eddy Currents - Finite Elements Extension

# Course Overview (1/3): Block 1/3

	Electromagnetism	Math
week 1	Electrostatics	1D Finite Difference Method
week 2	Three Applications (Ferreira)	
week 3	Electrostatics	2D Finite Difference Method
<b>Assignment on FDM for Electrostatics</b>		

# Course Overview (2/3): Block 2/3

week 4	Magnetostatics	1D Finite Element Method
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week 5	Magnetostatics	2D Finite Element Method
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<b>Assignment on FEM for Magnetostatics</b>
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<b>Meet and Great in /Pub</b> <b>Wednesday, March 2nd, 16:00 -17:00</b>
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# Course Overview (3/3): Block 3/3

week 6-7	Q.-Stat. Magnetics	2D FEM Extensions
	Skin Effect	hp-Adaptivity
		Local refinement
		Saturation
		Current Sheets
		Permanent Magnets
Assignment on FEM for Quasi-Stationary Magnetics		
Oral Exam		

# Overall Study Goals for this Course

- block 1/3: build your **own** 1D and 2D finite difference code
- block 2/3: build your **own** 1D and 2D finite element code
- block 3/3: give **interpretation** of finite difference and finite element simulation results

# Course Assessment

- $G_1$ ,  $G_2$  and  $G_3$ : grade for assignment
- $G_4$ : grade for oral exam
- final grade =  $(G_1 + G_2 + G_3 + 2G_4)/5$

# Course Preliminaries

- calculus of a function of one and two variables: (partial) derivative, integration over interval and surface
- linear algebra: linearly independent sets, basis, matrix-vector multiplication, linear system
- programming skills in Matlab or Python

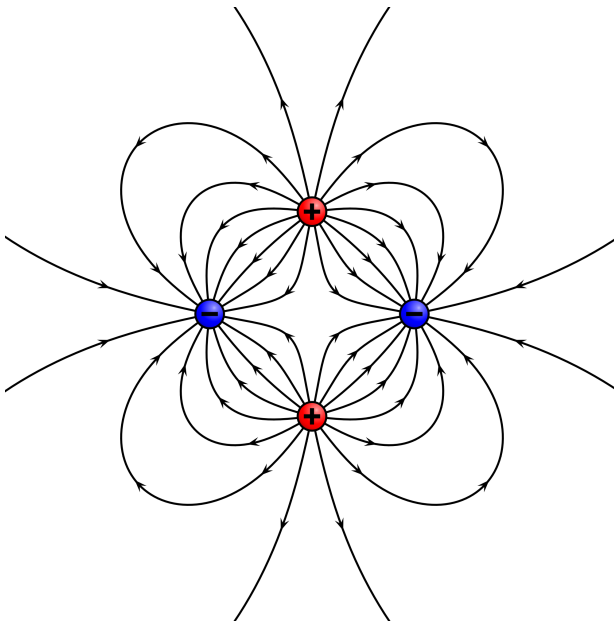


# Course Deadlines

- assignment  $G_1$ : hand in before March 4th, 2016 (week 4)  
2D Electrical Field of a Dipole
- assignment  $G_2$ : hand in before March 18th, 2016 (week 6)  
1D Magnetic Field of a Fault Current Limiter
- assignment  $G_3$ : hand in before April 15, 2016  
To be announced
- oral exam  $G_4$ : make appointment prior to April 30th

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# Block 1/3: Part 1/3: Electrostatics

## Study Goals for Electrostatics (1/2)

- motivate introduction of the potential  $\phi$  for the electric field  $\mathbf{E}$
- give definition of gradient of a scalar field, .e.g.  $\nabla\phi$
- give definition of divergence of a vector field, e.g.  $\nabla \cdot \nabla\phi$
- derive electrostatic field equations from the Maxwell equations and constitutive equations: decoupling of electric and magnetic field
- write down the Poisson equation for the electric potential
$$\nabla \cdot \sigma \nabla \phi = \rho$$
- diffusion coefficient  $\sigma$ : electrical conductivity
- source term  $\rho$ : spatial charge distribution

# Block 1/3: Part 1/3: Electrostatics

## Study Goals for Electrostatics cont'd (2/2)

- state and give interpretation to the Dirichlet boundary conditions (insulation)
- state and give interpretation to the Neumann boundary conditions (symmetry)
- post process the potential for the electrical field  $\mathbf{E}$  (and force)
- give motivating example: plate capacitor with and without end effects and with and without dielectricum

# Block 1/3: Part 2/3: Mathematical Preliminaries

- **calculus**: derivative, difference quotient, finite difference approximation  
Reference: book calculus
- **linear algebra**: vector, matrix, linear system  
Reference: book linear algebra

# Block 1/3: Part 3/3: 1D/2D Finite Difference Method

## Study Goals 1D Finite Difference Method

- write down 1D Poisson equation in the independent variable  $0 \leq x \leq 1$ :  $-u''(x) = f(x)$ . Give physical interpretation.
- write down Dirichlet boundary conditions  $u(x = 0) = 0$  and give physical interpretation
- write down Neumann boundary conditions  $u'(x = 1) = 0$  and give physical interpretation
- give physical interpretation of mathematical problem stated
- discretize the Poisson equation an internal grid point
- write down the finite difference stencil
- handle non-homogeneous Dirichlet and Neumann boundary conditions
- form matrix (discrete differential operator) and right-hand side vector (discrete source term)
- solve linear using backslash in Matlab
- compare numerically computed and analytically given solution

# Block 1/3: Part 3/3: 1D/2D Finite Difference Method

## Study Goals 2D Finite Difference Method

- give definition of gradient, divergence and Laplacian
- write down 2D Poisson equation in the independent variables  $0 \leq x, y \leq 1$ :  $-\Delta u(x) = f(x)$ . Give physical interpretation.
- write down Dirichlet and Neumann boundary conditions and give physical interpretation
- give physical interpretation of mathematical problem stated
- discretize the Poisson equation an internal grid point
- write down the finite difference stencil
- handle non-homogeneous Dirichlet and Neumann boundary conditions
- form and solve linear using backslash in Matlab
- compare numerically computed and analytically given solution



# Block 1/3: Timeline

## Week 1

- 2 hours of lectures and 2 hours of lab sessions
- Lecture 1: 1D Finite Difference Method
  - first 15min: introduction to this course
  - next 15min: Math Preliminaries on Calculus: first and second derivative of a function in one variables  $u(x)$  and its finite difference discretization
  - last 15 min: Linear Algebra: matrix-vector multiplication and linear system solve
- Lecture 2: 1D Finite Difference Method (cont'd):
  - problem formulation
  - internal point discretization, boundary treatment, linear system formulation
  - comparison numerical and analytical solution
- Lab session: 1D Finite Difference Method in Matlab or Python

# Block 1/3: Timeline

## Week 2

- by Braham Ferreira

# Block 1/3: Timeline

## Week 3

- 2 hours of lectures and 2 hours of lab sessions
- Lecture 1: 2D Finite Difference Method
  - problem formulation
  - internal point discretization
- Lecture 2: 2D Finite Difference Method (cont'd)
  - boundary treatment
  - linear system formulation
  - comparison numerical and analytical solution
- Lab session: 2D Finite Difference Method in Matlab or Python

# Block 1/3: Assignment

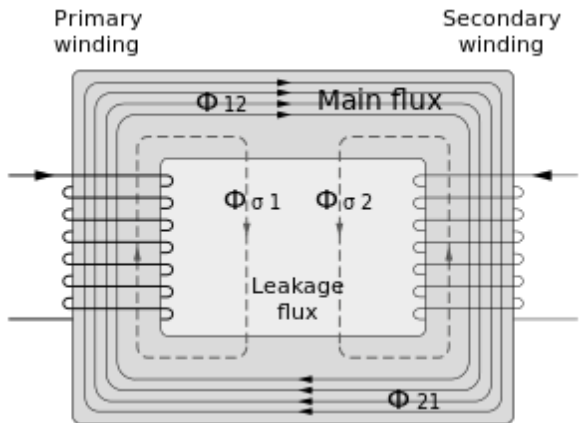
- assignment: simulate dipole (and quadrupole?) field using 2D FDM code. Report on methodology and results obtained. In discussing results, mention effect of mesh and boundary conditions employed.

## Block 1/3: Lab Sessions - Possible Extensions

- spatially varying diffusion coefficient (basic model for saturation)
- different number of cells along coordinate axes
- adding a zeroth order term (basic model for eddy currents)
- adding a convective term (basic model for motion)
- extension to 3D

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# Block 2/3: Part 1/2: Magnetostatics

- scalar and vector potential formulation
- boundary conditions
- derivation from Maxwell equations
- example of the magnetic density separator



## Block 2/3: Part 1/2: Magnetostatics

- magnetic scalar potential model

$$-\frac{\partial}{\partial x} \left( \mu \frac{\partial u(x, y)}{\partial x} \right) - \frac{\partial}{\partial y} \left( \mu \frac{\partial u(x, y)}{\partial y} \right) = f_{\mu}(x, y)$$

supplied with boundary conditions

- magnetic vector potential model

$$-\frac{\partial}{\partial x} \left( \frac{1}{\mu} \frac{\partial u(x, y)}{\partial x} \right) - \frac{\partial}{\partial y} \left( \frac{1}{\mu} \frac{\partial u(x, y)}{\partial y} \right) = f_{1/\mu}(x, y)$$

supplied with boundary conditions

## Block 2/3: Part 2/2: 1D/2D Finite Element Method

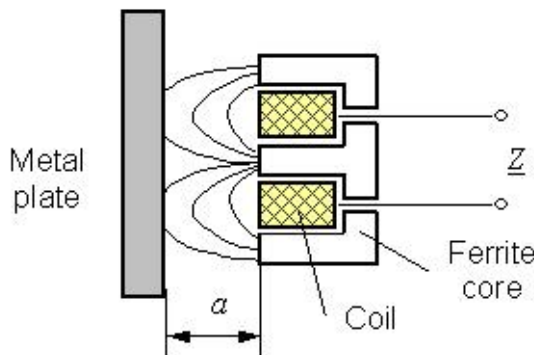
- math preliminaries: integration by parts, quadrature and function spaces
- first (and second) order Lagrangian FEM on triangles
- continuous and spatially discrete weak or variational formulation
- elementary matrix and right-hand side
- element-by-element matrix and right-hand side assembly
- linear system solve
- compare with analytical model problem and finite difference approximation
- 1D, 2D square and 2D irregular domain
- interpreting results

## Block 2/3: Lab Sessions - Assignment

- week 3: develop 1D FEM code and compare with analytical solution
- week 4: develop 2D FEM code and compare with analytical solution
- assignment: apply 1D FEM code to one-dimensional model of the fault current limiter

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## Block 3/3: Part 1/2: Quasi-Stationary Magnetics

- extension of model equations discussion to include eddy-current effects
- analytical solution for model problem that includes eddy current effect
- extension to cylindrical coordinates to model axi-symmetrical configurations
- extension to non-linear effects to include saturation - Picard and Newton (?) linearization
- extension to include current sheet: line-by-line assembly to include line current sources
- extension to include permanent magnets modeled as opposite current sheets
- extension to both scalar and vector potential formulation to describe Hallbach magnet arrays

## Block 3/3: Part 2/2: Extension on 2D FEM

- extension of element-by-element assembly to include mass matrix contribution
- extension to second order elements to increase accuracy of the computation
- include local mesh refinement to capture skin effects

## Block 3/3: Lab Sessions - Assignment

- week 5: extend 2D FDM code to non-unit square domain with non-constant coefficients: coil-core-air configurations. First stationary, then quasi-stationary (ask Henk for suggestions?).
- week 6: extend 2D FDM code to coil-core-magnet-air configurations: linear actuator or Hallbach array
- week 7: extend 2D FEM code to saturation and/or second order elements
  
- assignment: variation of one of the above