Application of the algebraic flux correction algorithm to a one-dimensional water flood model

Supervisors: Matthias Möller (TUD) and Johan Romate (Shell/TUD)

Problem background

In petroleum reservoir engineering water flooding is a technique to enhance the oil recovery from a reservoir. In water flooding water is injected in one or more places (injection wells) under high enough pressure for the oil to be pushed by the injected water towards the producing wells.

Here, we consider water flooding in one space dimension. On one end water is injected and on the other end oil and water are produced. We assume the oil and the water to be incompressible. The two-phase flow model of incompressible fluid flow through a porous medium in one space dimension we consider, is given by the transport equations for the phase masses (w=water, o=oil),

$$\frac{\partial}{\partial t}(\phi\rho_o S_o) + \frac{\partial}{\partial x}(\phi\rho_o v_o) = 0, \tag{1}$$

$$\frac{\partial}{\partial t}(\phi\rho_w S_w) + \frac{\partial}{\partial x}(\phi\rho_w v_w) = 0, \tag{2}$$

and Darcy's equations,

$$q_o = -\frac{kk_{ro}}{\mu_o}\frac{dp}{dx} = -\lambda_o\frac{dp}{dx}, \tag{3}$$

$$q_w = -\frac{kk_{rw}}{\mu_w}\frac{dp}{dx} = -\lambda_w\frac{dp}{dx}. \tag{4}$$

The actual velocities v_i are related to the Darcy (or superficial) velocities q_i by $q_i = \phi v_i$. Additionally, we require that the saturations add to one:

$$S_o + S_w = 1. \tag{5}$$

Here, the reservoir is taken horizontal: the effect of gravity is neglected. The densities ρ_o and ρ_w are constant, as are the porosity ϕ, the absolute permeability k and the viscosities μ_o and μ_w. The pressure gradient $\frac{dp}{dx}$ is not necessarily constant, but we do assume that the water injection rate is constant, and thus that the total velocity $q_T = q_o + q_w$ is constant.

The relative permeabilities k_{ro} and k_{rw} are functions of the water saturation, using $S_o = 1 - S_w$,

$$k_{ro} = k_{ro}^0\left(\frac{1 - S_w - S_{or}}{1 - S_{wc} - S_{or}}\right)^{no} \tag{6}$$

$$k_{rw} = k_{rw}^0\left(\frac{S_w - S_{wc}}{1 - S_{wc} - S_{or}}\right)^{nw}. \tag{7}$$

Assignment

In [2] and [3], the algebraic flux correction (AFC) methodology is described for the Galerkin finite element method applied to scalar conservation laws and hyperbolic systems, respectively. In essence, AFC constitutes a modern approach based on rigorous algebraic construction principles for the design of high-resolution

schemes both for time-dependent (FCT-type approach) and stationary problems (TVD-type approach). In this assignment the applicability of this methodology on the above one-dimensional water flood model (1)–(7) will be investigated. This includes a comparison with existing methods for solving this model.
As part of the assignment a literature study has to be carried out on FCT-type methods for finite element and finite volume methods in one and multiple space dimensions. Special attention should be paid to their distinguishing features, the requirements on the models and numerical methods, and the properties of the resulting solutions. The assignment consists of the following parts:

1. Literature study.
2. Description of the model and development of the numerical method.
3. Setting up and testing the resulting numerical model.
4. Investigation and comparison with existing model(s).
5. Writing the thesis.

Literature

[1] K. Aziz; A. Settari: *Petroleum Reservoir Simulation* Applied Science Publishers, London, 1979.

[2] D. Kuzmin: Algebraic Flux Correction I: Scalar Conservation Laws.
In: D. Kuzmin; R. Löhner; S. Turek (eds.) *Flux-Corrected Transport: Principles, Algorithms, and Applications.* Second Edition. Springer, 2012.

[3] D. Kuzmin; M. Möller; M. Gurris: Algebraic Flux Correction II: Compressible Flow Problems.
In: D. Kuzmin; R. Löhner; S. Turek (eds.) *Flux-Corrected Transport: Principles, Algorithms, and Applications.* Second Edition. Springer, 2012.