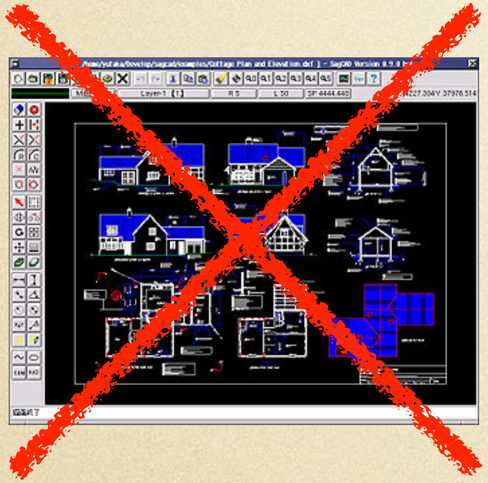
The Future is Analog, *maybe*

Matthias Möller, Delft University of Technology, NL

Antoni Gaudi

1852-1926

1852-1926



1852-1926

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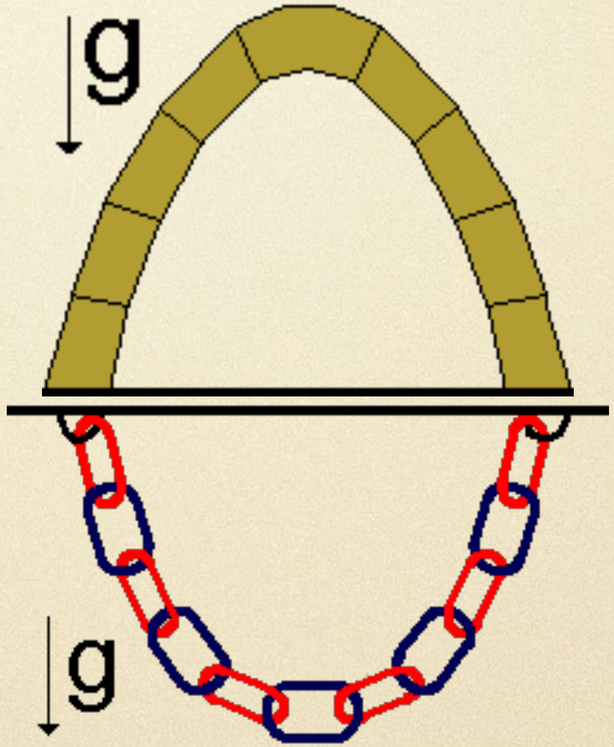
Gaudi's hanging chain model



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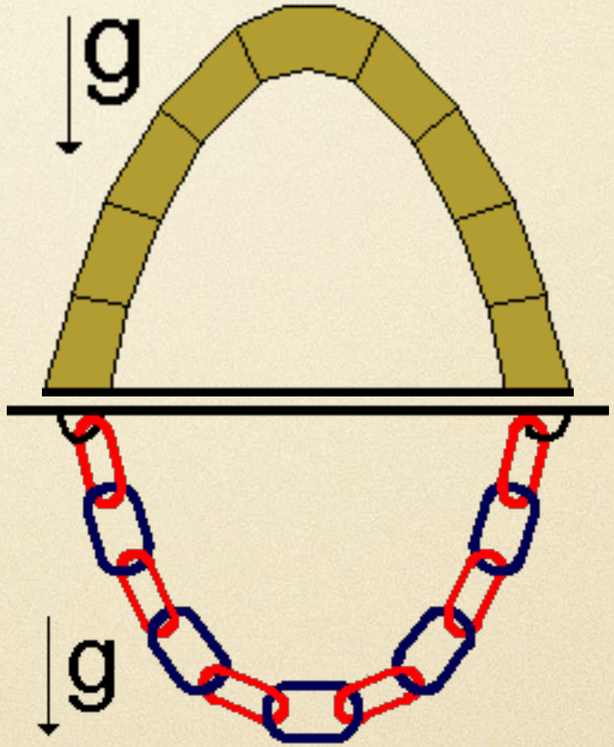

I ♥ analog computing

I ♥ numerics

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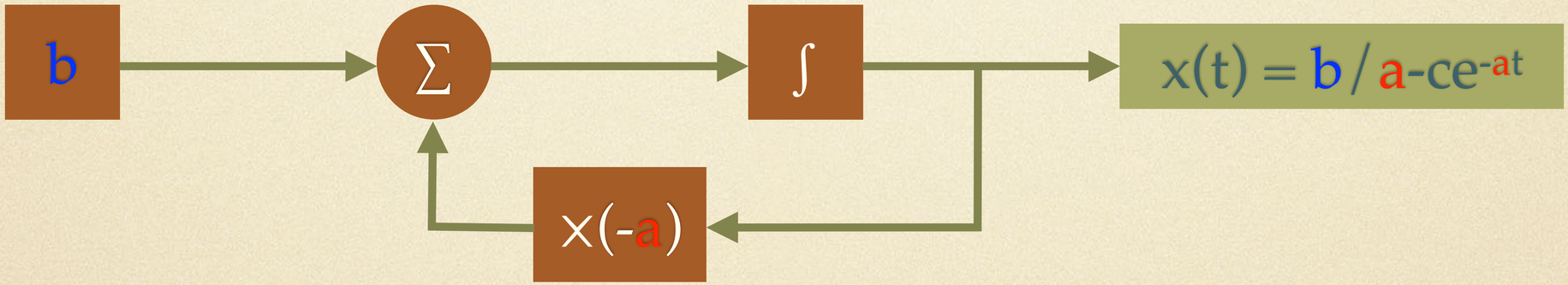
Can I solve my problems by analog computing?

Let's try to solve an ODE ...

$$\dot{x}(t) = b - a \cdot x(t)$$

$$x(0) = x_0$$

… with an analog integrator



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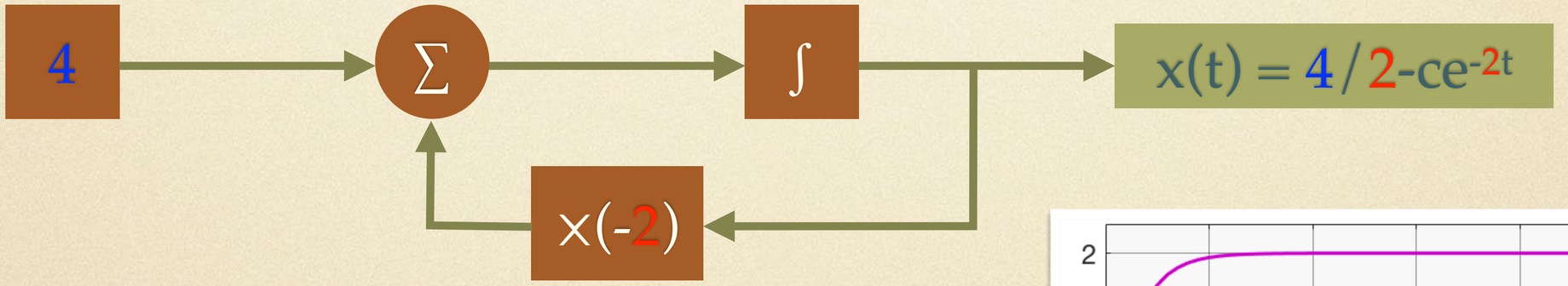

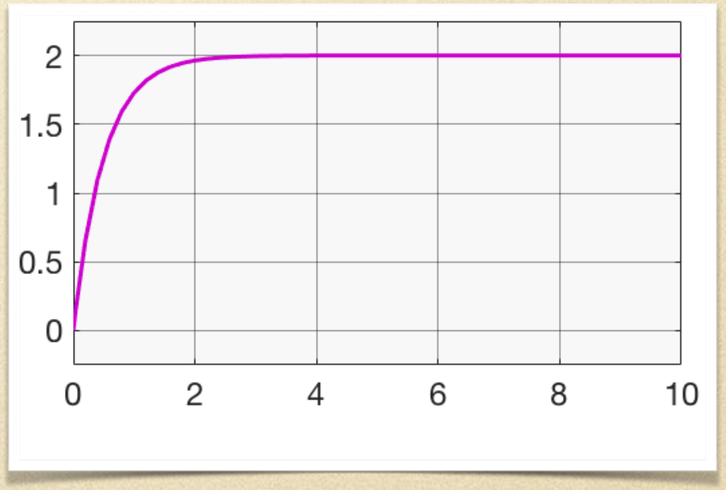

How about linear systems?

$$A \cdot X = B$$

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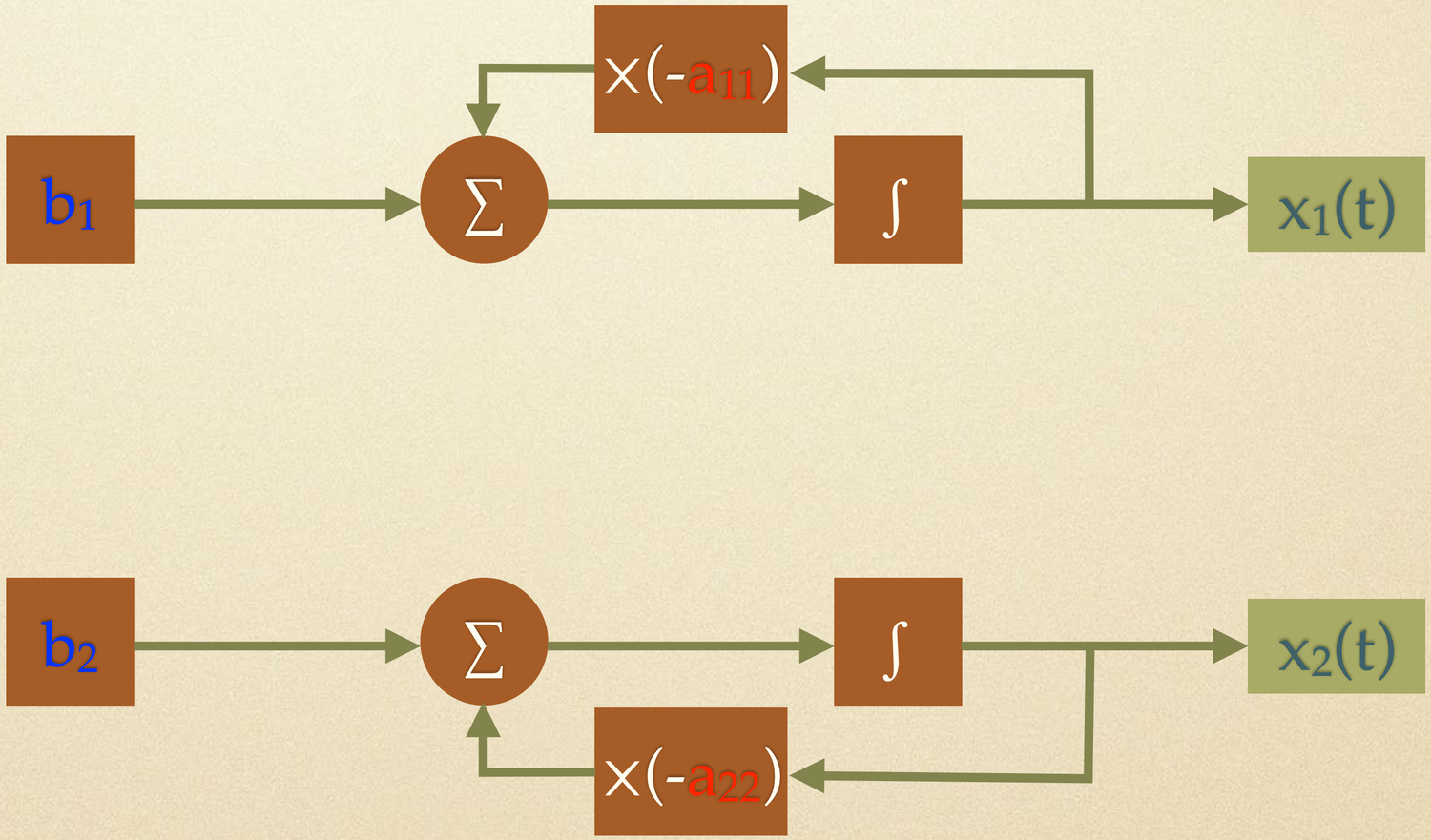
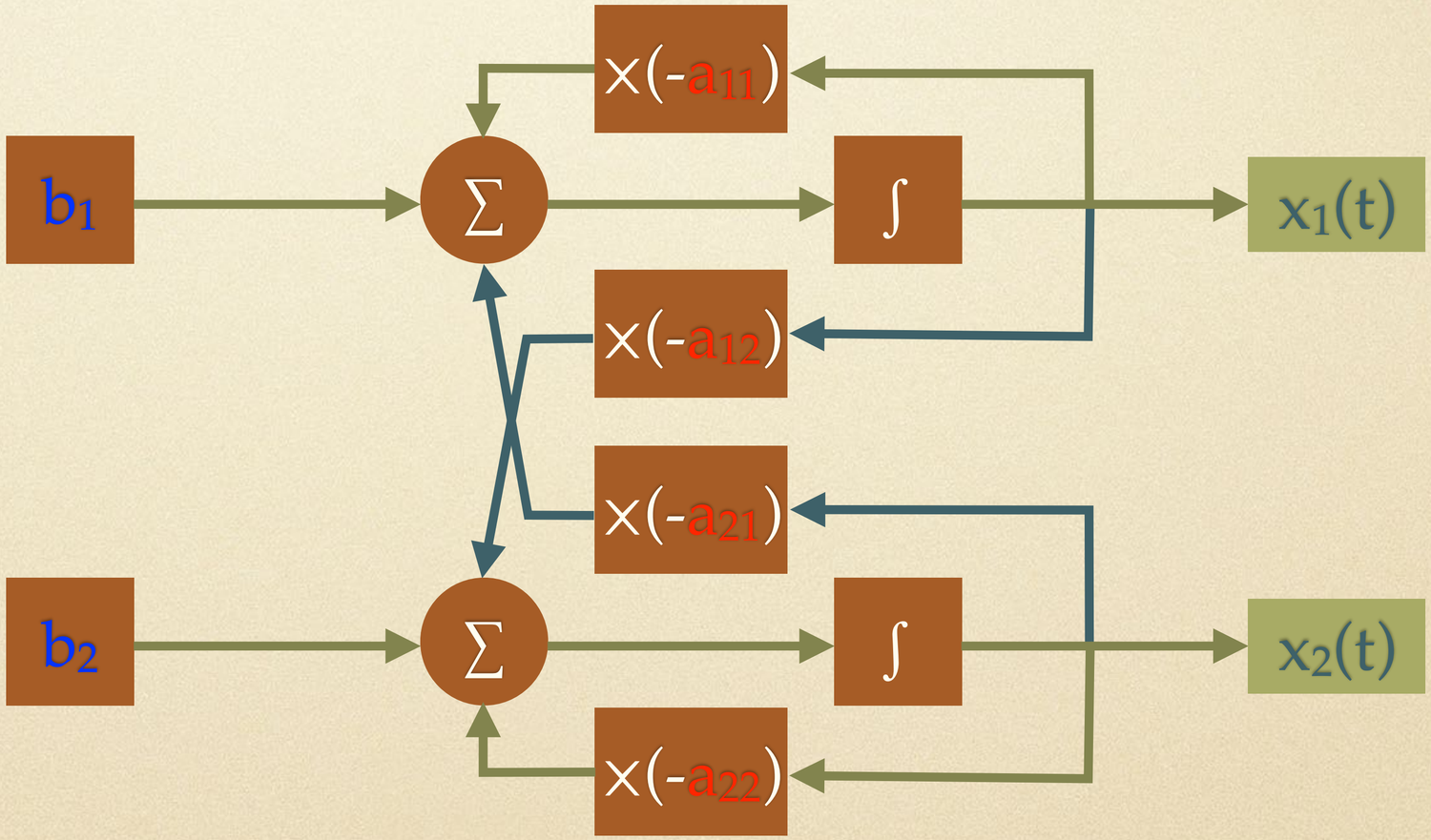
$$X(0) = X_0$$


x(-a11)
b1
Σ
∫
x1(t)
b2
Σ
∫
x2(t)
x(-a22)

Et voilà!



Now try it in practice!

$$1\,x_1 - 2\,x_2 = 1$$

$$-2\,x_1 + 1\,x_2 = -1$$

$x_1 = \frac{1}{3} \qquad x_2 = -\frac{1}{3}$

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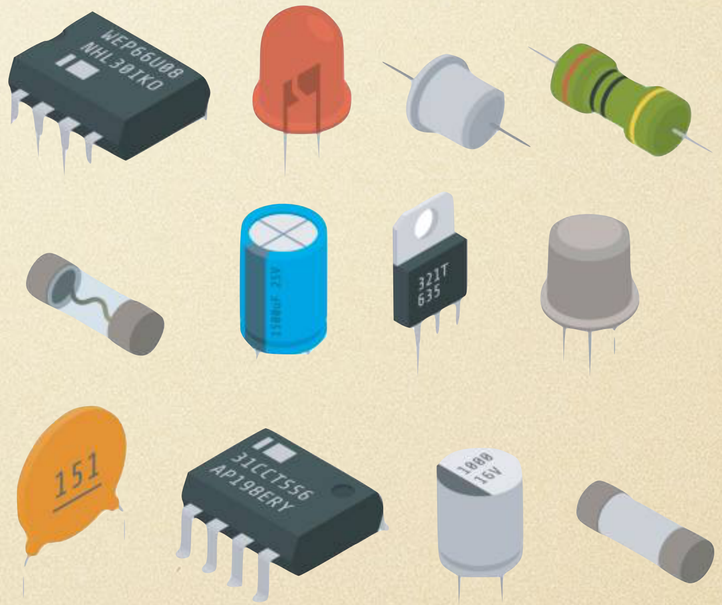

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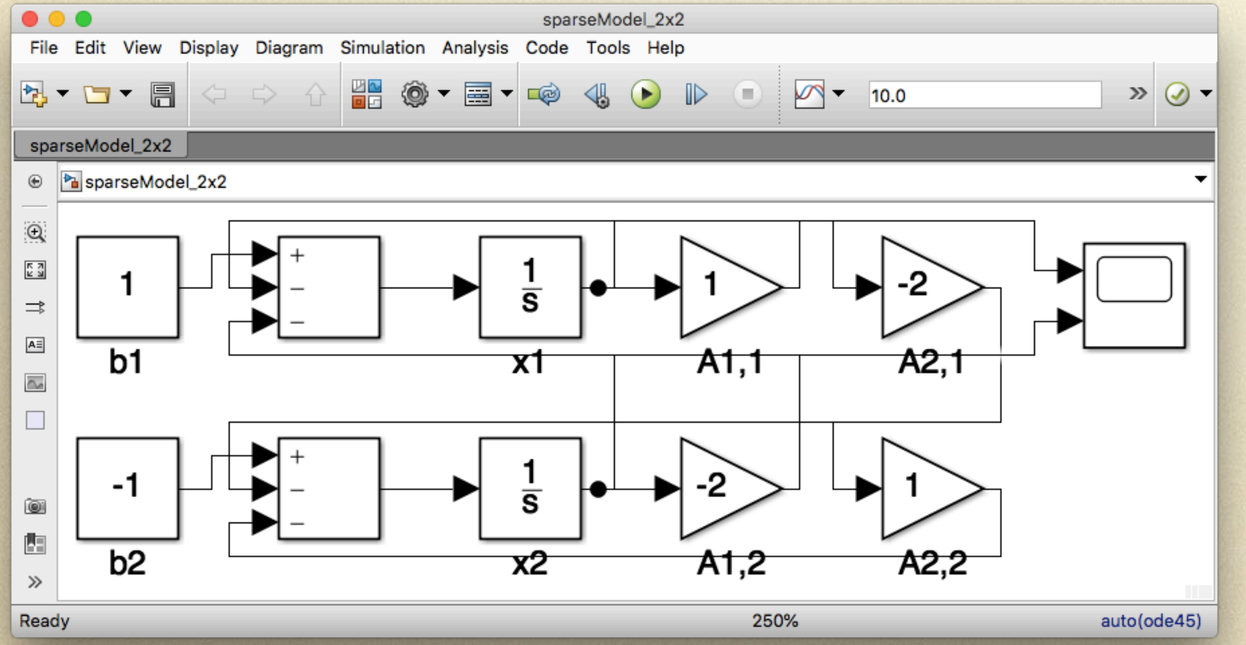
Better simulate it!



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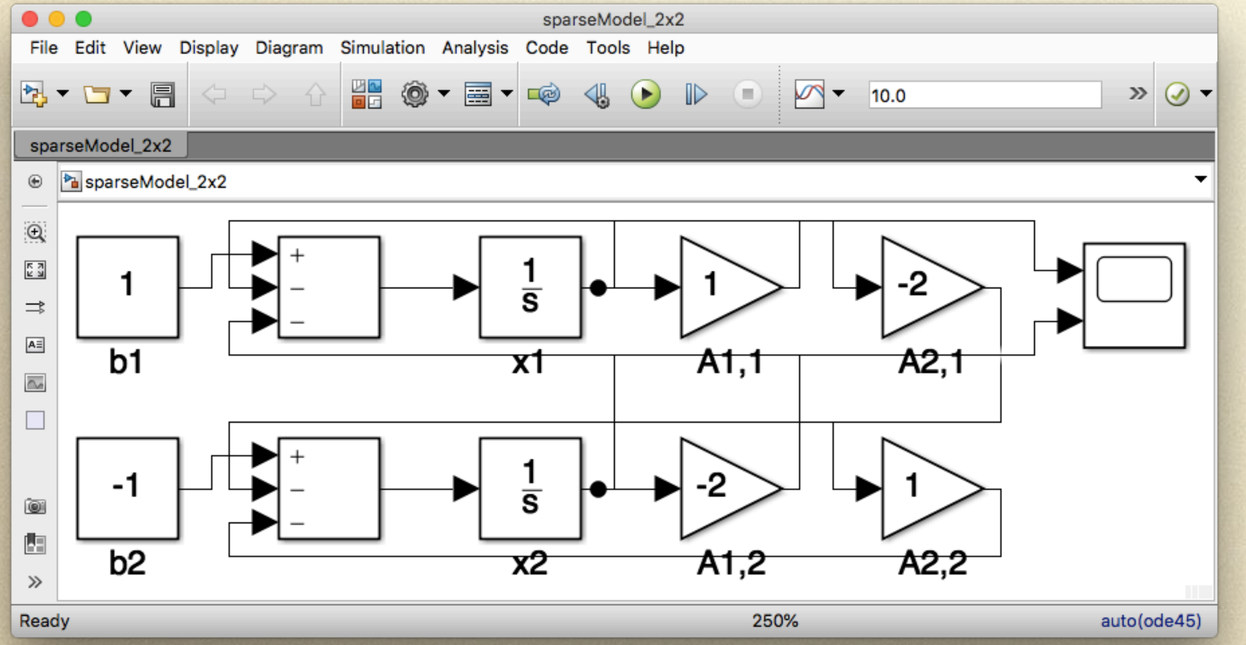

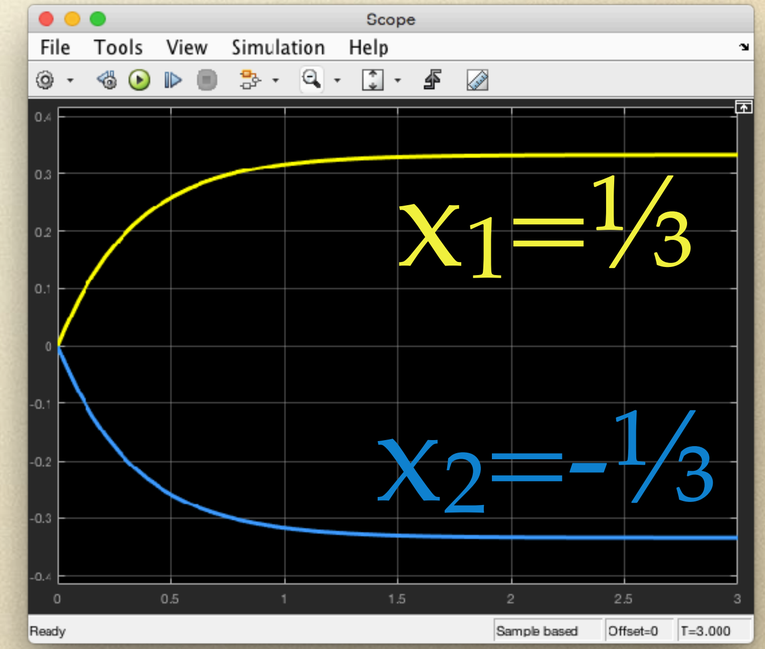

Convergence may take some time!

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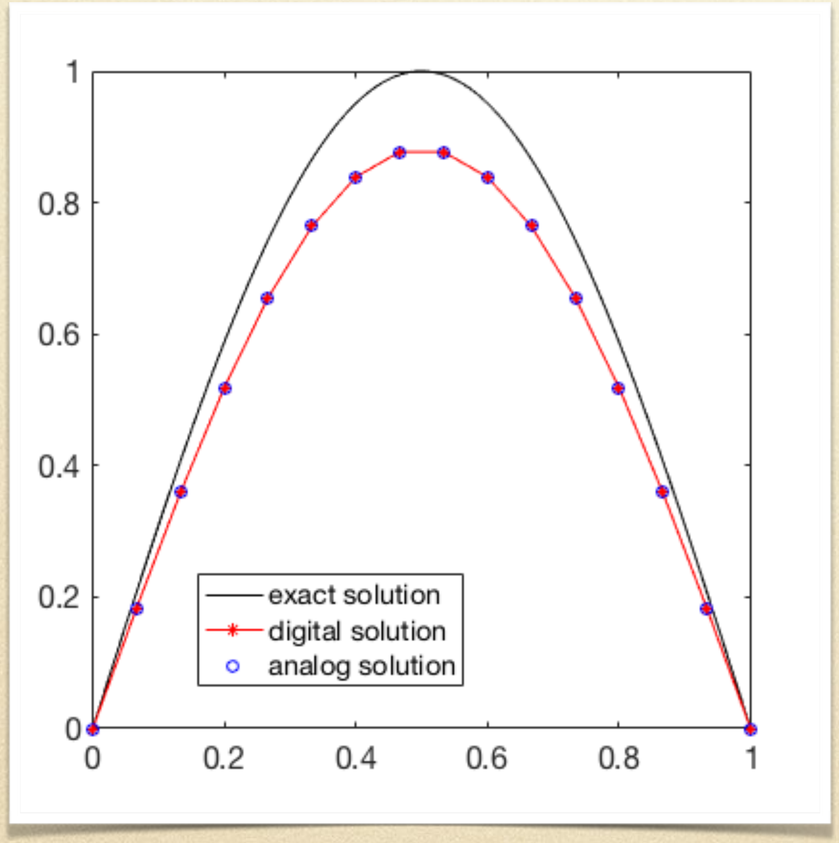
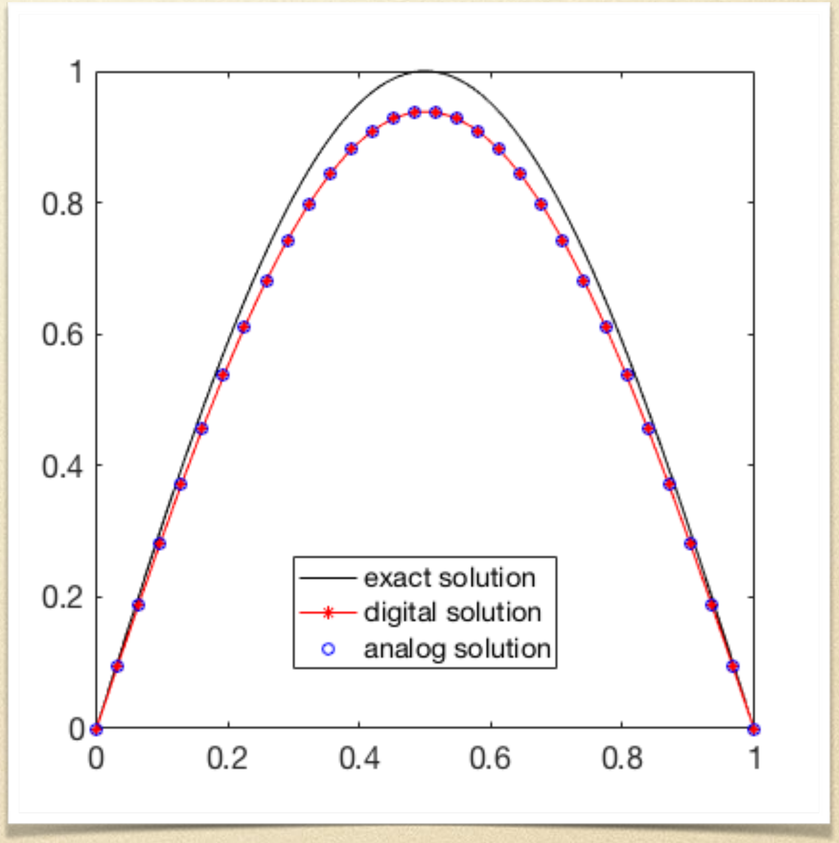
I know!

$$-u''(x) = f(x)$$

Finite differences

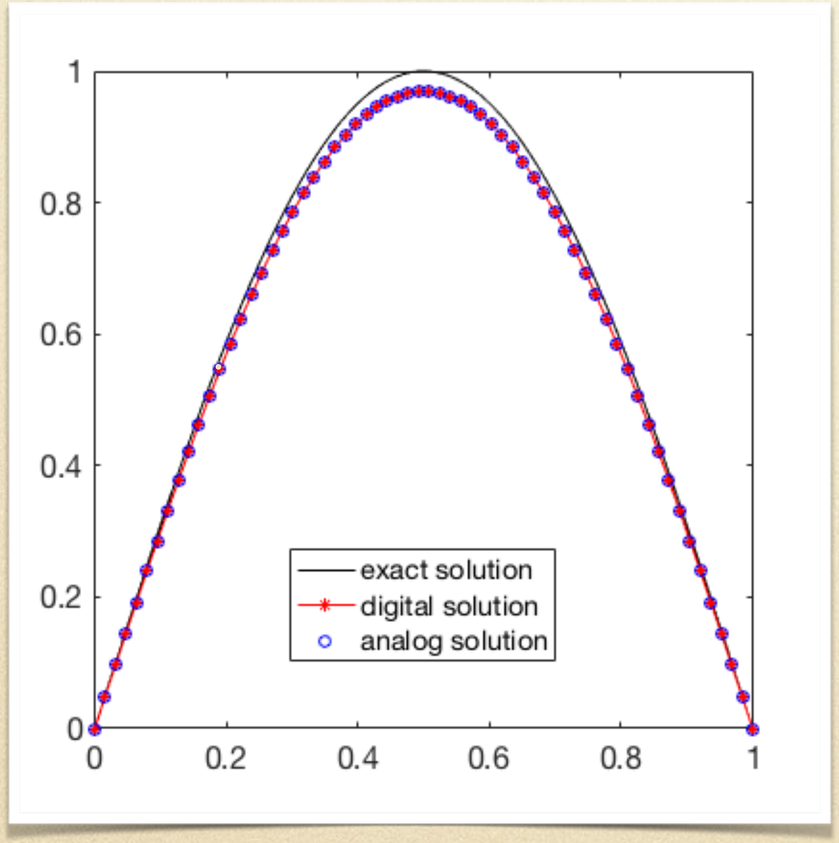


16 unknowns

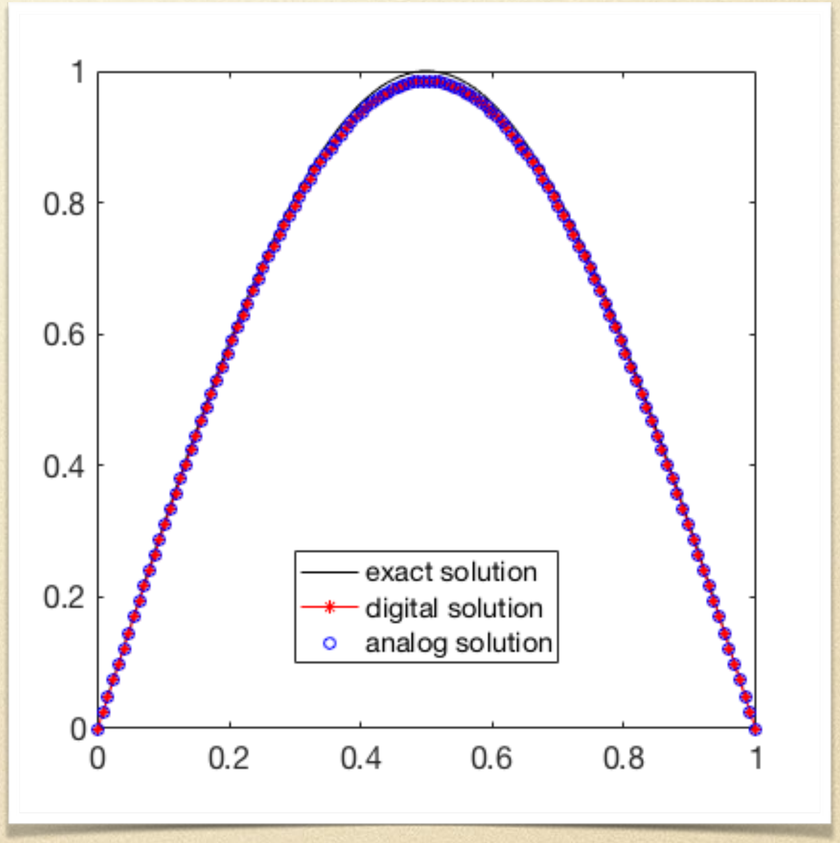


32 unknowns

Finite differences



64 unknowns



128 unknowns

$$-u''(x) = f(x)$$

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Let's do spectral finite elements!

It all boils down to:

Diagonal matrix entries

$$a_{ii} = i\, i\, \pi^2 \int \cos^2(i\pi x)\, dx$$

Vector entries

$$b_i = \int \sin(i\pi x)\, f(x)\, dx$$

Solution

$$u(x) = \sum_j u_j \sin(j\pi x)$$

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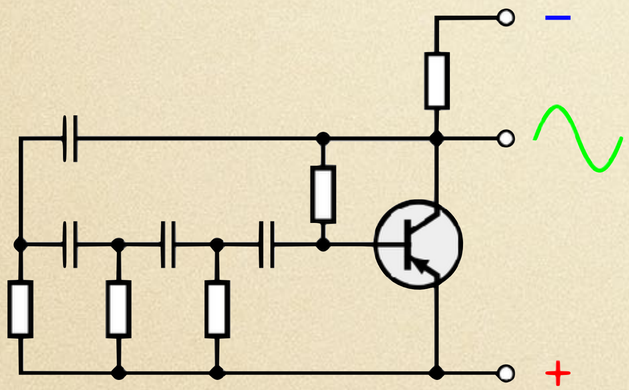

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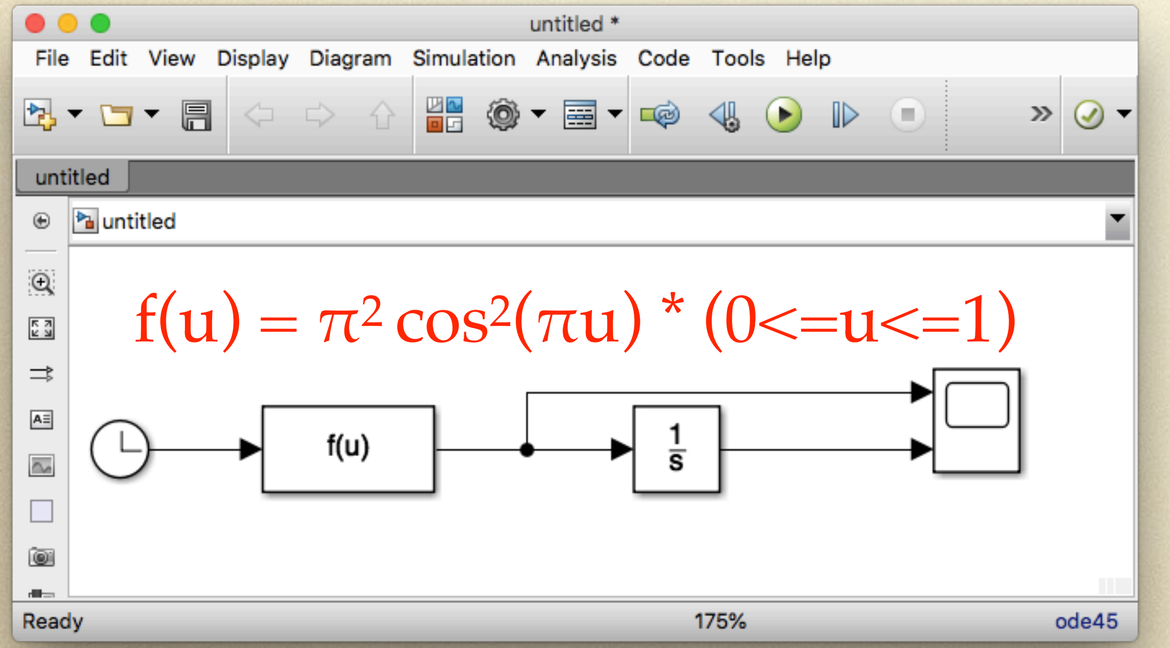
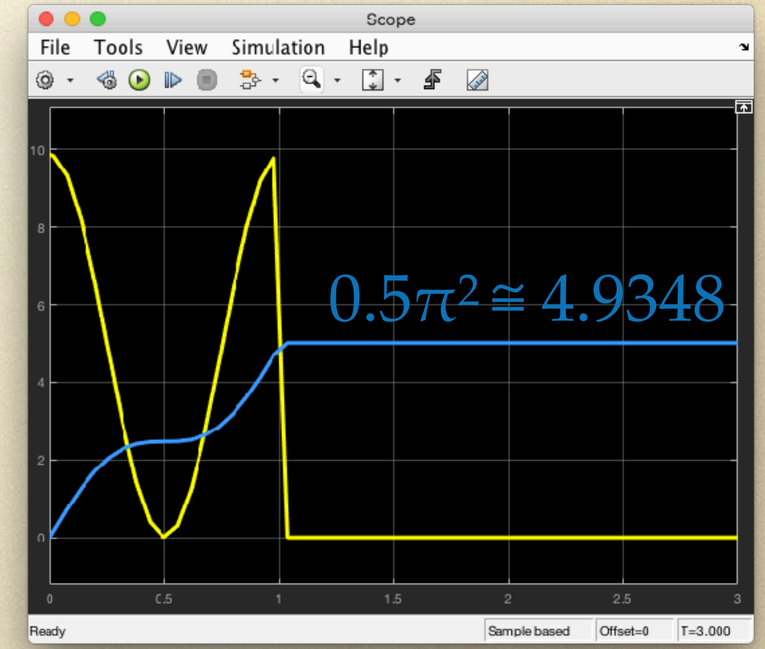
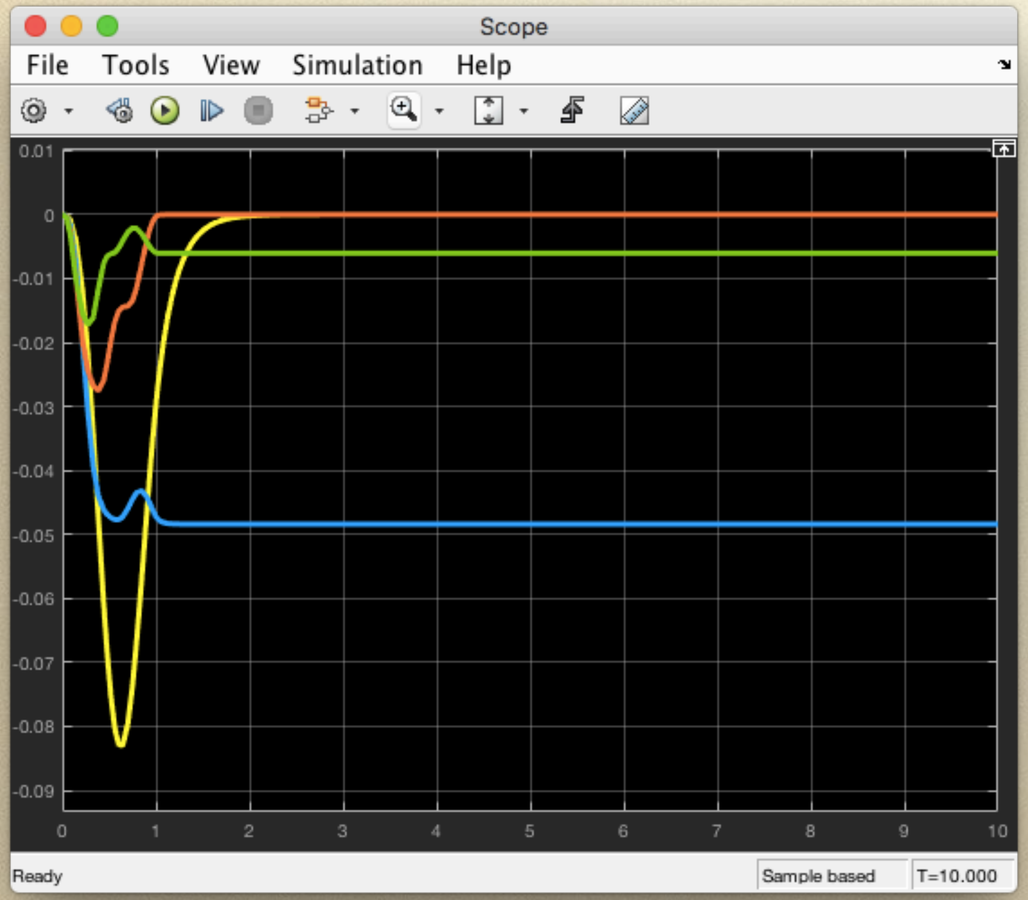
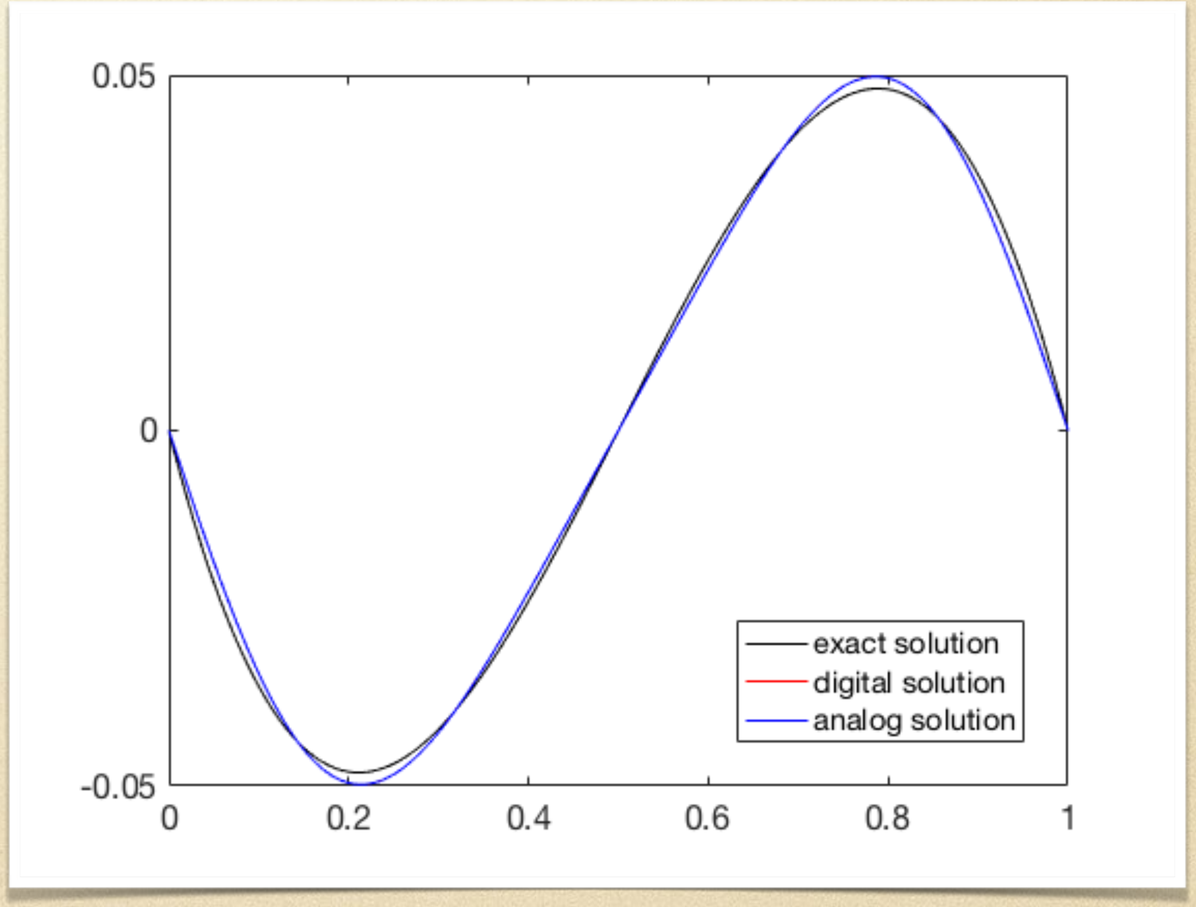
Function integrator



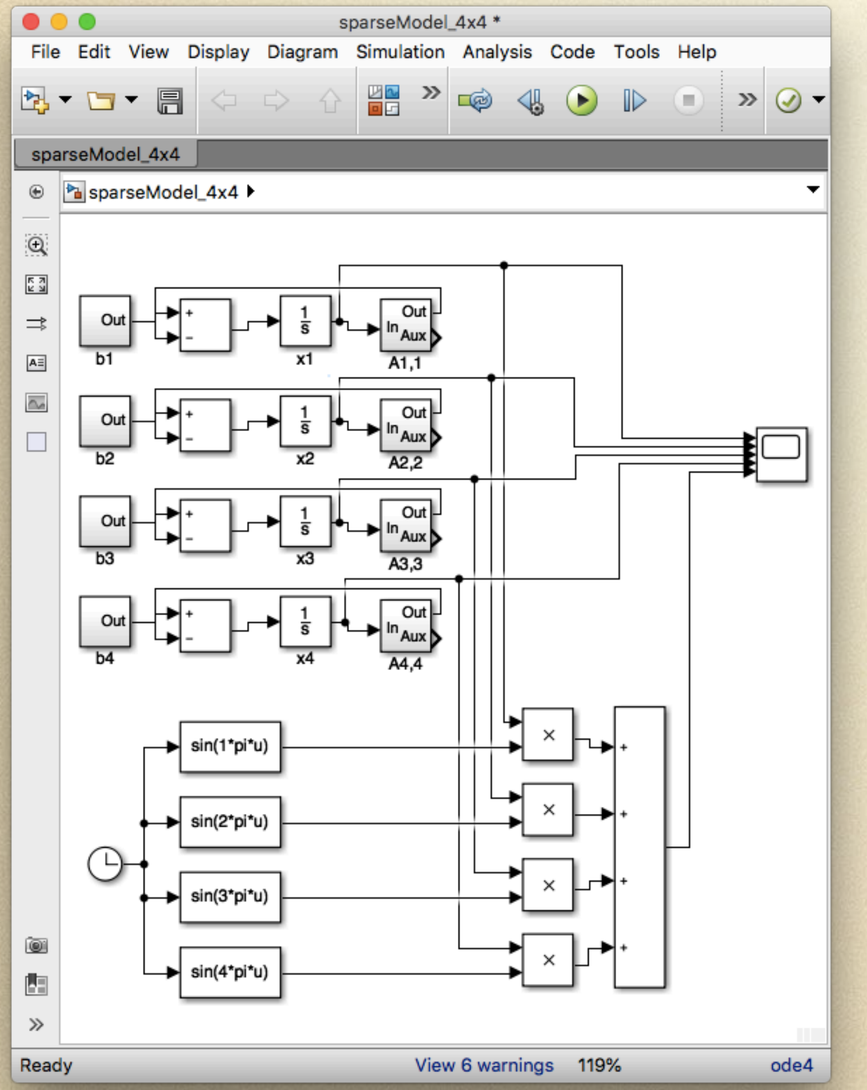


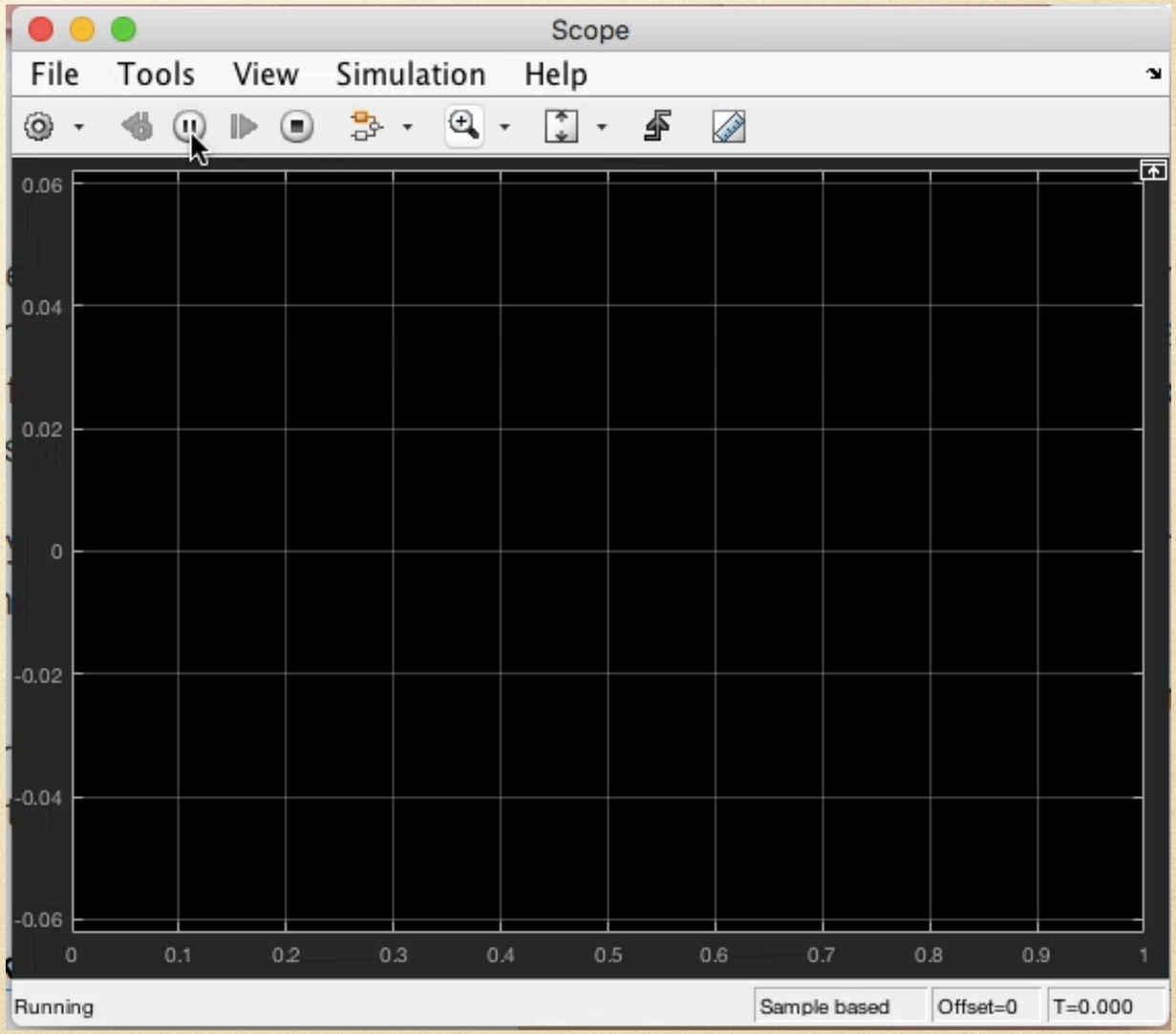
SPECFEM1D_Analog





But wait, we can use a scope





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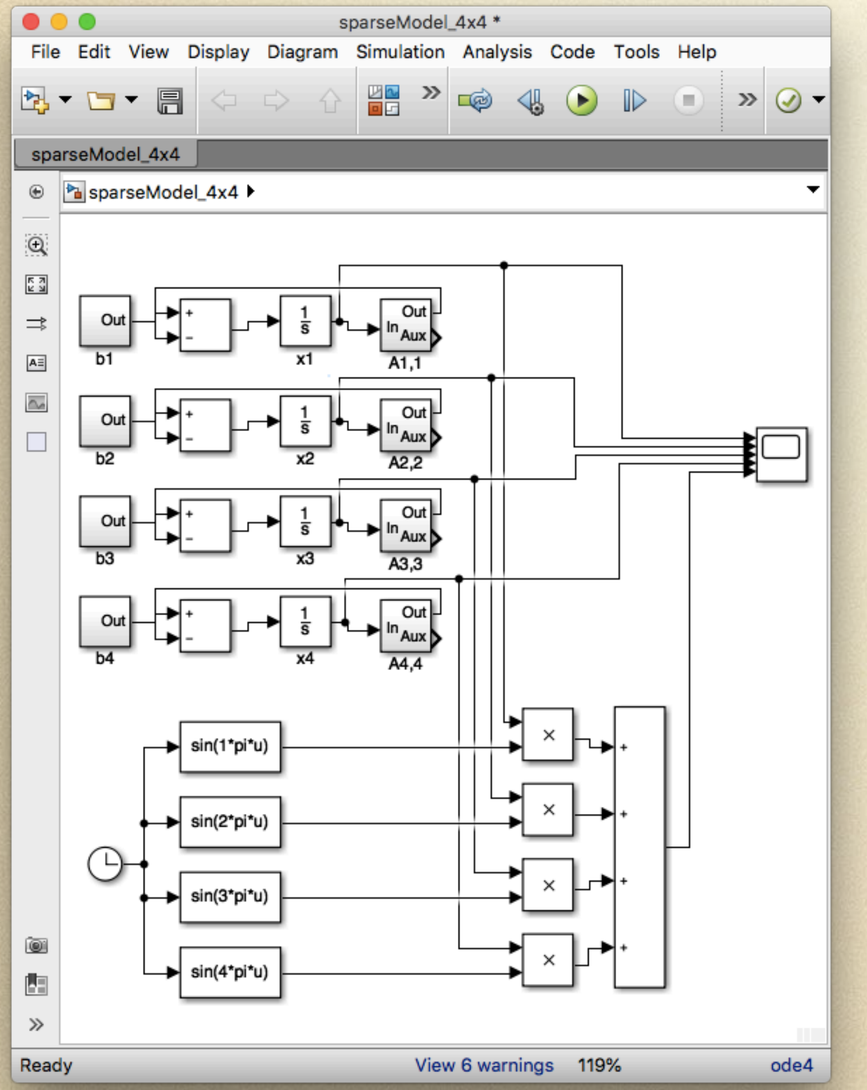
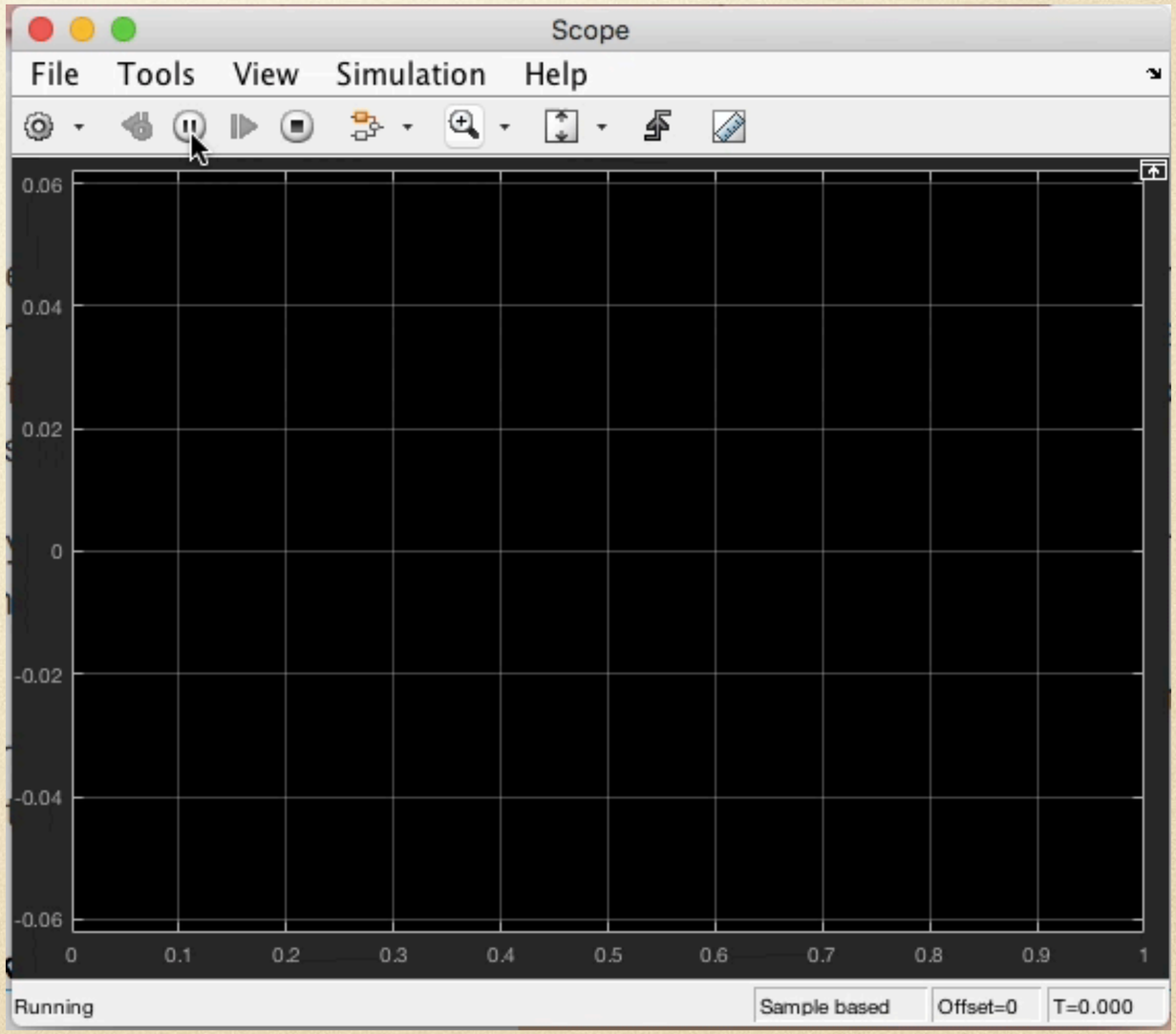
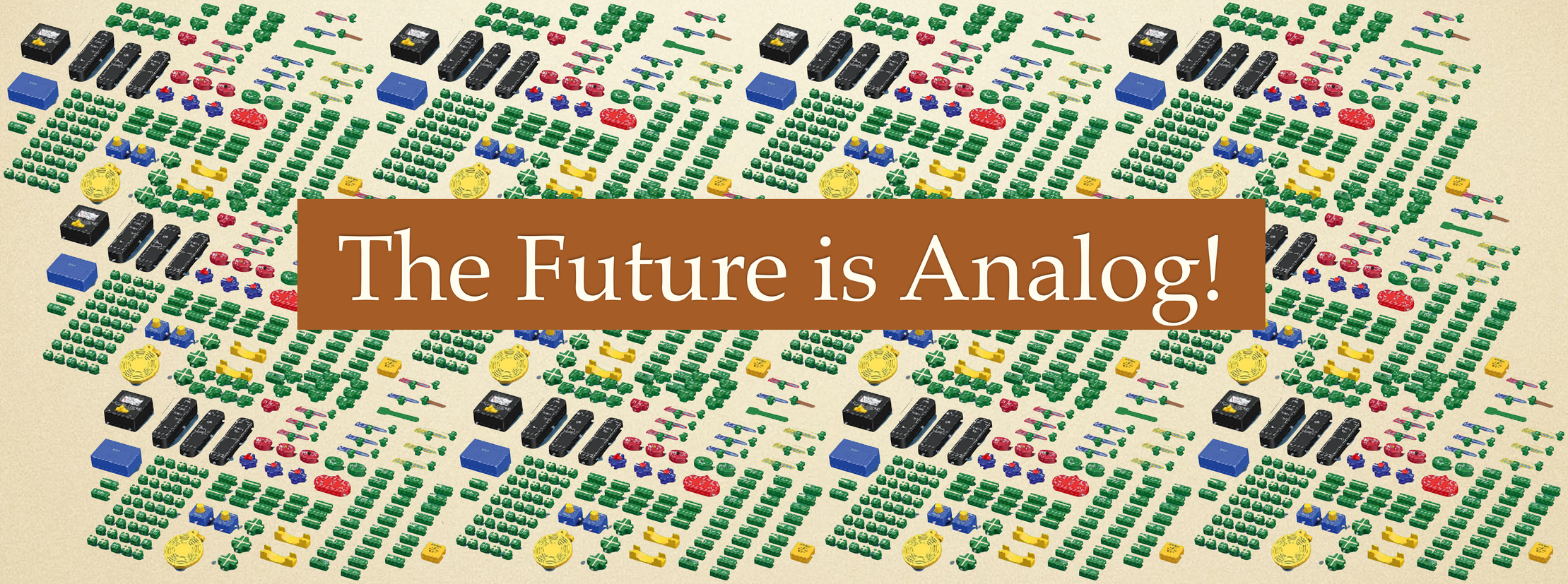




ANALAB© on sale soon



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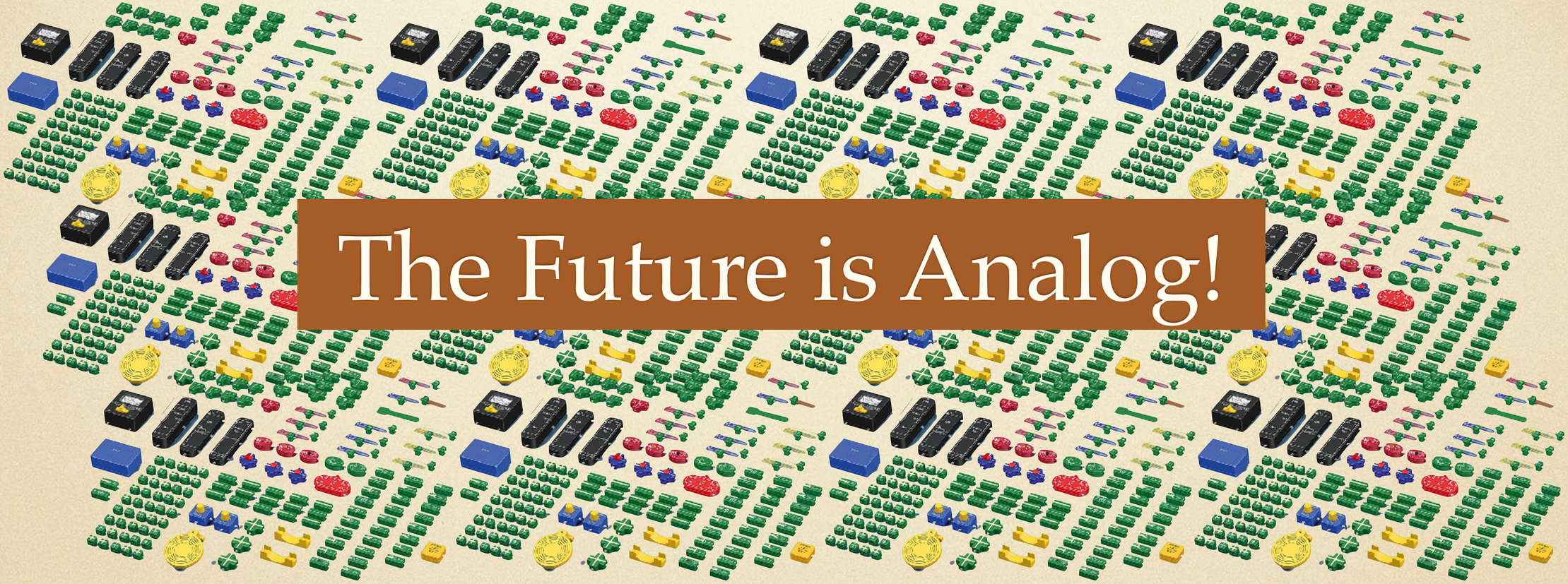

Thanks for your attention and keep soldering.