Multiwavelet troubled cell indicator for discontinuity detection

Thea Vuik[*], **Jennifer Ryan**

[*]M.J.Vuik@tudelft.nl

Introduction

In this poster, we introduce a new global troubled cell indicator for the discontinuous Galerkin (DG) method. Here, the global DG approximation is re-expanded in terms of a multiwavelet basis, which is a sum of a global average and finer details on different levels. Examining the higher level difference coefficients acts as a troubled cell indicator. This indicator is able to reduce the computational cost by avoiding limiting in smooth regions.

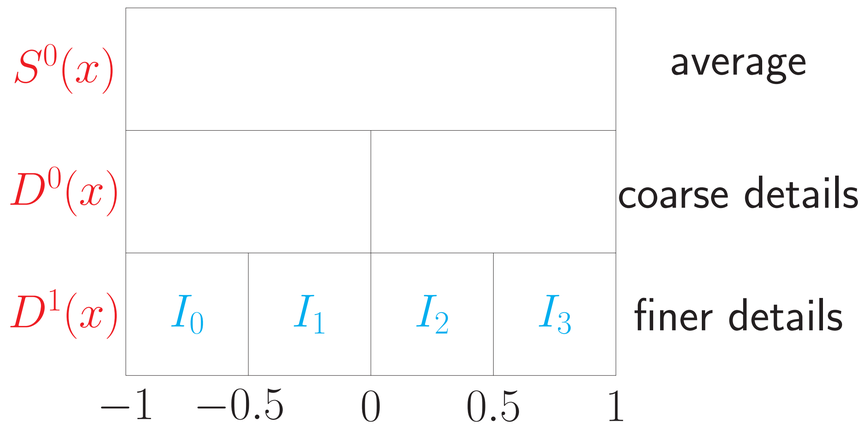
Ideas

- Example: uses 2^n elements on $[-1, 1]$ $(n = 2)$
  - Global DG approximation of degree k:

$$u_h(x) = \sum_{j=0}^{3} \sum_{\ell=0}^{k} u_j^{(\ell)} \phi_\ell(\xi_j)$$

  - Corresponding multiwavelet decomposition:

$$u_h(x) = S^0(x) + \sum_{m=0}^{n-1} D^m(x), \; n-1 = 1.$$



Continous regions, multiwavelet contributions

- Troubled cells: focus on highest level $D^{n-1}(x)$
  - Compute absolute average $\bar{D}_j^{n-1}$ on element I_j
  - Element I_j is a troubled cell if,

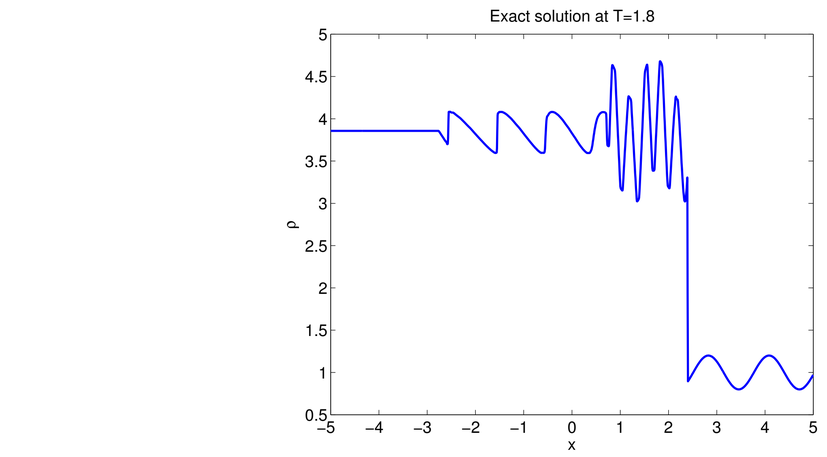
$$\bar{D}_j^{n-1} \geq C \cdot \max\left\{\bar{D}_i^{n-1}, i = 0, \ldots, 2^n - 1\right\},$$

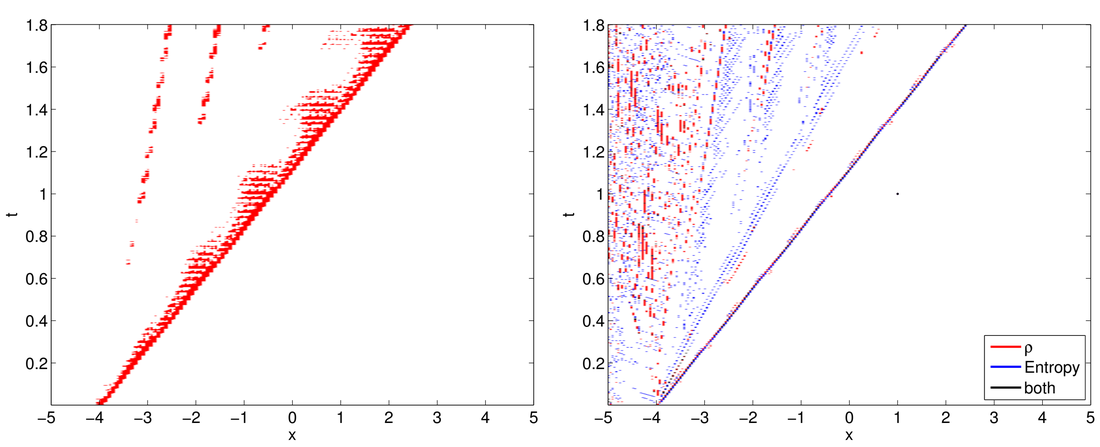
$C \in [0, 1]$

- Parameter C: defines strictness of indicator,
  - $C = 0$: every element is detected
  - $C = 0.2$: select largest 80% of averages
  - $C = 0.8$: select largest 20% of averages
- Global detector, more accurate than local detector
- Use troubled cell indicator as switch in moment limiter

Results for Euler equations

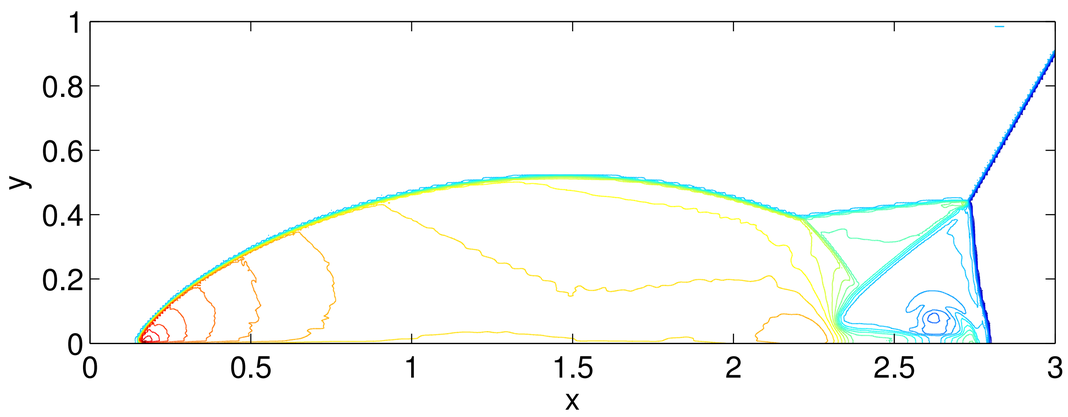
- Sine-entropy wave: multiwavelet indicator (density as indicator variable) and Harten's indicator (density and entropy)

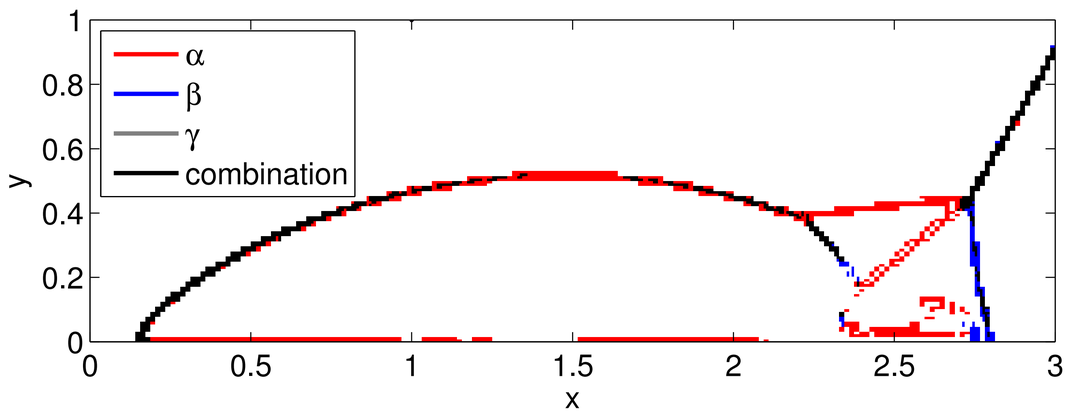




Indicated troubled cells using $C = 0.1$ (left) or Harten (right)

- Double Mach reflection: detection in different modes





Solution (above) and indicated troubled cells (below) at $T = 0.2$, $C = 0.05$

Conclusion

We constructed a new global multiwavelet troubled cell indicator, using density as an indicator variable. The results look promising.

TU Delft Delft University of Technology

Delft Institute of Applied Mathematics