Automated parameters for troubled-cell indication via outlier detection

Thea Vuik, Jennifer Ryan

DG method for hyperbolic problems

Consider the PDE $u_t + f(u)_x = 0$ with initial and boundary conditions. An approximation for the solution can be found using the discontinuous Galerkin method:

$$u_h(x,t) = \sum_{\ell=0}^{k} u_j^{(\ell)}(t)\phi_\ell(\xi_j),\ j = 0,\ldots,2^n-1,$$

where ξ_j is the local variable $2/\Delta x(x - x_j)$.

Troubled-cell indicators for DG

Solutions of nonlinear hyperbolic PDEs usually contain shocks or develop discontinuities in time. In order to avoid spurious oscillations a limiter can be applied. Detection of discontinuities is done by applying a troubled-cell indicator, for example:

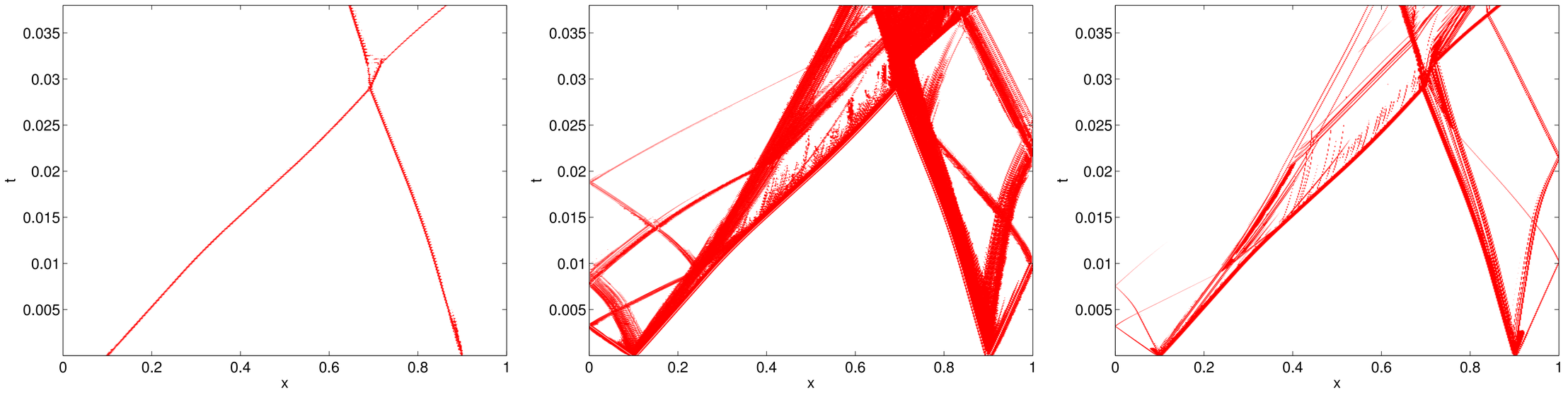
- Multiwavelet troubled-cell indicator: detect I_j and I_{j+1} if
$$|d_{kj}^{n-1}| > C \cdot \max\{|d_{kj}^{n-1}|, j = 0,\ldots,2^n-1\},$$
- KXRCF shock detector: detect I_j if
$$\frac{\left|u_h|_{I_j} - u_h|_{I_{n_j}}\right|}{\left(\frac{\Delta x}{2}\right)^{\frac{k+1}{2}}|\bar{u}_j|} > 1, \quad j = 0,\ldots,2^n-1,$$
- Minmod-based TVB indicator: detect I_j if
$$\left|\sum_{\ell=1}^{k} u_j^{(\ell)}\phi_\ell(1)\right| > M\Delta x^2 \text{ or } \left|\sum_{\ell=1}^{k} u_j^{(\ell)}\phi_\ell(-1)\right| > M\Delta x^2,$$
$j = 0,\ldots,2^n-1.$

How to choose parameters?

Outlier detection using boxplot idea

Send in a suitable troubled-cell indication vector $\mathbf{D}$.
Split this vector into local vectors, $\mathbf{d}$.
for all local vectors **do**
  Sort $\mathbf{d}$ to obtain $\mathbf{d}^s$.
  Compute the quartiles Q_1 and Q_3.
  Determine outliers:
$$d_j \notin [Q_1 - 3(Q_3 - Q_1), Q_3 + 3(Q_3 - Q_1)].$$
end for
Check detected outliers in left and right half of local region with neighboring regions.

Parameters no longer problem dependent!



(a) Multiwavelet troubled-cell indicator, $C = 0.05$ (b) KXRCF shock detector, threshold 1 (c) Minmod-based TVB indicator, $M = 100$

(d) Outlier, multiwavelet troubled-cell indicator (e) Outlier, KXRCF shock detector (f) Outlier, minmod-based TVB indicator

Example: one-dimensional Euler equations for simulation of interaction of two blast waves.
Time-history plot of detected troubled cells, original indicator (top) or outlier-detected approach (bottom), $k = 3$, 512 elements.

TU Delft Delft University of Technology

Delft Institute of Applied Mathematics