Material to know for Numerical methods for differential equations (wi3097)

For all subjects we assume that you know the definition/expression and you are able to use these concepts.

1. Introduction numerical analysis
   - Taylorpolynomial and remainder term
   - Big O symbol and computational rules
   - Definition absolute and relative error
   - Floating point numbers and rounding error
2. Interpolation
   - Linear interpolation formula, Lagrange interpolation, if the formulas are given you should be able to use them.
   - Remainder term for Linear and Lagrange interpolation
   - Be familiar with the definition of a spline. If the formulas are given you should be able to construct a cubic spline.
3. Numerical differentiation
   - Forward, Backward and central difference for the first order derivative
   - Be able to derive general difference formulas for first order and higher order derivatives.
   - derivation of the truncation and rounding error
   - Richardson error estimate
4. Non-linear equations
   - Bisection and fixed point method, know the formulas and be able to use them
   - Termination criterion for bisection and a linear convergent process, derivation and application
   - Be able to give a graphical convergence/divergence plot of the fixed point iteration
   - Newton Raphson method know the formulas and be able to use them, including graphical interpretation and local convergence
   - Be able to derive the quadratic convergence of the Newton Raphson method
   - Use of the Newton Raphson method for non-linear systems
   - Application: non-linear boundary value problem
5. Numerical integration
   - Rectangular rule and Trapezoidal rule, know the formulas and be able to use them
   - For both methods you are able to derive the truncation and rounding error behavior.
   - For both methods you can give the composite rule and the remainder term of the composite rule.

6. Initial value problems
    - Forward Euler, Backward Euler, Implicit Trapezoidal rule, Modified Euler
    - If the formulas of RK4 are given you are able to use them
    - explicit and implicit
    - derivation of local truncation error
    - Order global error = order local truncation error effect of rounding errors
    - Efficiency comparison of various numerical methods
    - Stability, stable differential equation
    - Test equation, stability numerical method
    - Derivation of amplification factor $Q(h\lambda)$, investigate when $|Q(h\lambda)| \leq 1$
    - Analysis of the stability of a general differential equation
    - Stability numerical method for a general differential equation
    - If a numerical method is stable the the order of the global and local truncation error are the same
    - Be able to apply a numerical method to a system of differential equations
    - rewrite a higher order initial value problem to a system of first order differential equations
    - numerical stability of a (general) system of first order differential equations, use of a stability region
    - implicit methods are very suitable for stiff systems

7. Boundary value problems
    - Norm of a vector and the norm of a symmetric matrix
    - Condition number of a system of equations
    - Know the Gershgorin theorem and be able to use it
    - Be able to discretize a general second order boundary value problem with finite differences
    - Derive the local truncation error
    - Use of the boundary conditions and construction of the resulting linear system
    - Definition of stability, global error. If the method is stable, then the order of the global and local truncation error are the same
    - Use of a Neumann boundary condition and derivation of the local truncation error, use of a virtual point