Computation of Compressible and Incompressible Flows on Unstructured Staggered Grids

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1. Unstructured grids: motivation

- Easier grid-generation.
- Local (adaptive) refinement.

2. Staggered grids

a. Positioning of the variables

- Scalar quantities (ρ, ρH, p, $\ldots$) on centroids.
- Normal momentum component $m = (\mathbf{m} \cdot \mathbf{N})$, with $\mathbf{m} = \rho\mathbf{u}$, at midpoints faces.

b. Use of staggered and colocated grids

	Staggered	Colocated
Compressible	Our work	•
Incompressible	•	•

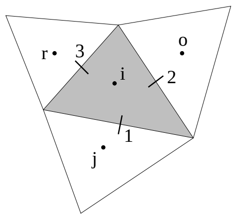
c. Advantages

- No spurious pressure oscillations in incompressible case.
- Fewer interpolations required in evaluation fluxes.
- Resulting scheme is simple.

d. Disadvantage

- No theory about mimicking hyperbolic character of Euler equations in discretization.

3. Discretization continuity equation



$$\int_{\mathrm{CV}} \frac{\partial \rho}{\partial t}\, d\mathbf{x} + \int_{\mathrm{CV}} \nabla \cdot (\rho \mathbf{u})\, d\mathbf{x} \approx$$
$$\approx \Omega_i \frac{\rho_i^{n+1} - \rho_i^n}{\Delta t} + \sum_e \rho_e u_e l_e = 0.$$

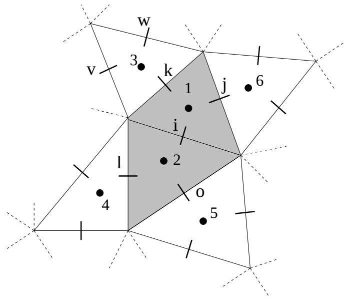
a. First order upwind

$$\rho_1 = \begin{cases} \rho_i & \text{if flow from } i \text{ to } j; \\ \rho_j & \text{if flow from } j \text{ to } i. \end{cases}$$

b. Central differences

$$\rho_1 = \frac{1}{2}(\rho_i + \rho_j).$$

4. Discretization momentum equation



a. Reconstruction procedure

$$\mathbf{N}_i = \eta_v \mathbf{N}_v + \eta_w \mathbf{N}_w.$$

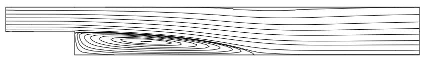
b. First order upwind

$$\mathbf{m}_k \cdot \mathbf{N}_i = \begin{cases} \eta_v m_v + \eta_w m_w & \text{if flow from 3 to 1;} \\ m_i & \text{if flow from 1 to 3.} \end{cases}$$

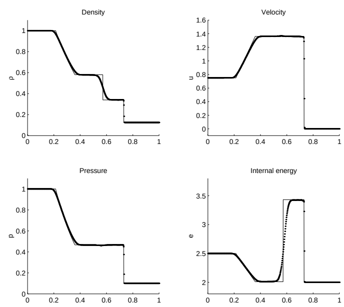
c. Central differences

$$\mathbf{m}_k \cdot \mathbf{N}_i = \frac{1}{2}(m_i + \eta_v m_v + \eta_w m_w).$$

5. Backward facing step (Re = 389)



6. Modified Sod problem



7. Transonic profile flow

NACA0012 profile, $M_\infty = 0.8$, $\alpha = 1.25^o$.