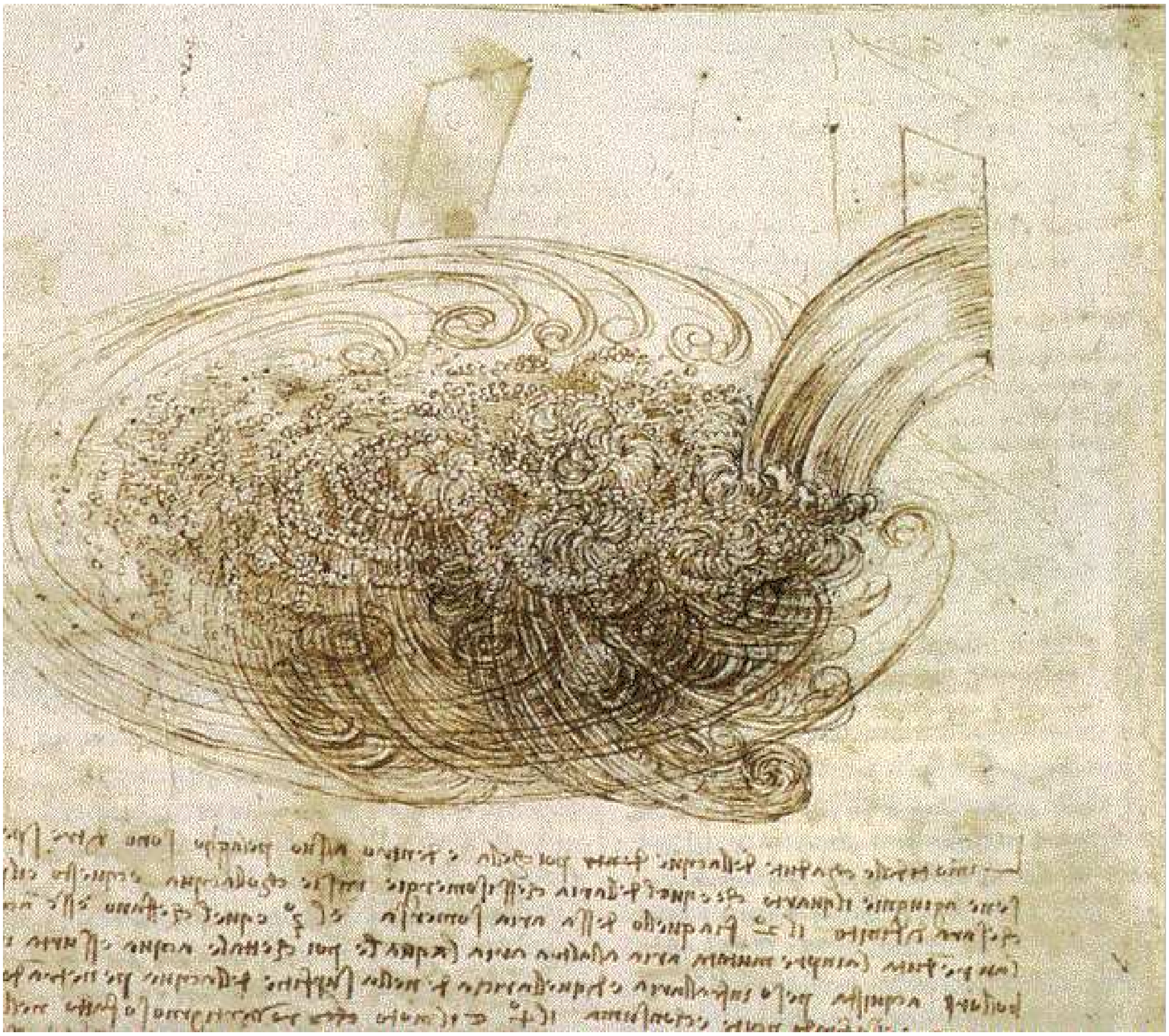
Turbulence: Modeling complex flow phenomena

C.W. Oosterlee



(Leonardo da Vinci ~ 1508)

Laminar and Turbulent

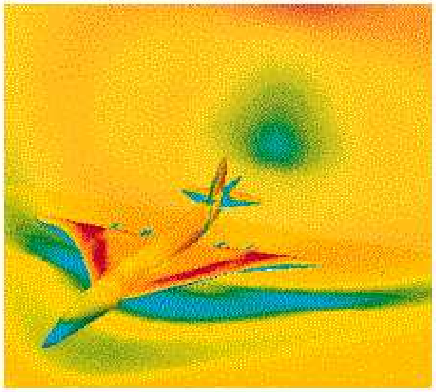
Laminar flows

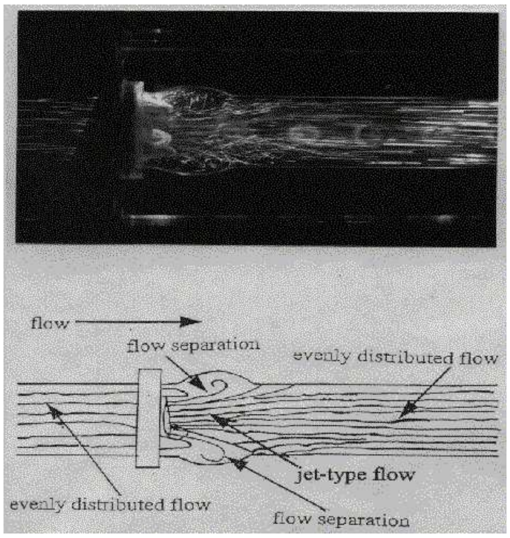
- are regular and smooth
- can be 1D, 2D, 3D and stationary
- are an exception !

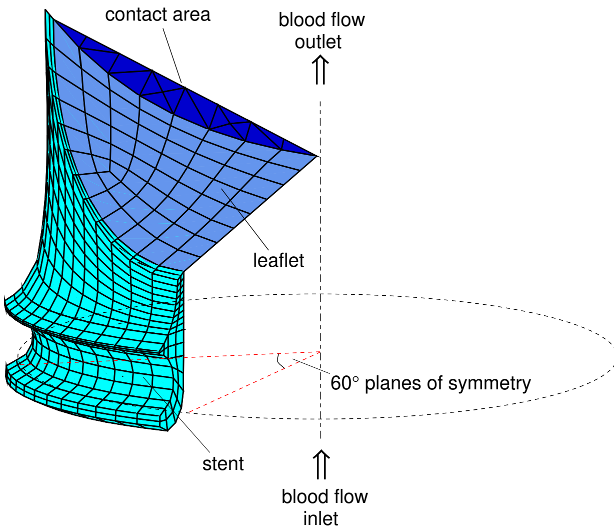
Turbulent flows are

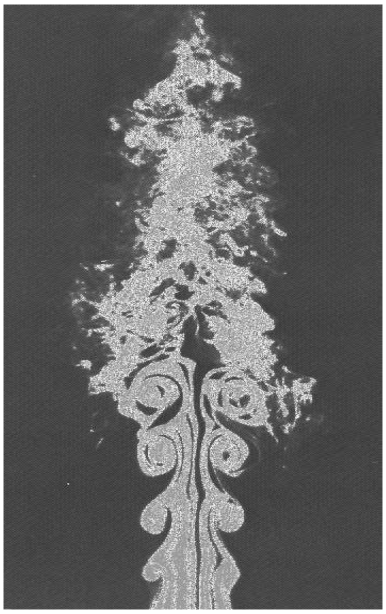
- full of rotation: eddies
  of all scales
- irregular: complicated variations of
  velocity, temperature etc.
- unsteady and 3D
- mixing
- dissipative: viscous forces operate on the smallest eddies,
  that transport kinetic energy into heat

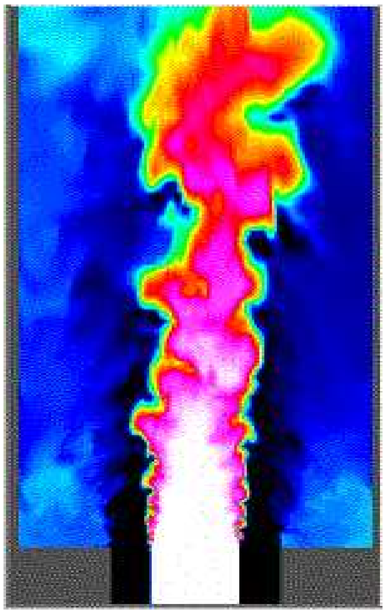
Examples of turbulent flow











Necessity for numerical simulation

- For optimization of designs, engineers have to predict flow phenomena

- Experiments on the real object are often impossible or very expensive

- Model experiments can be expensive and time consuming, certain flow phenomena cannot be simulated in a laboratory

  $\Rightarrow$ Computations are for certain flows the only way to gain insight, for other flows they are cheaper than experiments.

The mathematical equations

- Flow phenomena are usually described by the Navier-Stokes equations.

x-momentum:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial p}{\partial x} = \frac{1}{Re}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

$Re = \overline{U}L/\nu$ (Reynoldsnumber)

(similar for y- and z-momentum)

(Navier 1785-1835, Stokes 1819-1903)

continuity equation for an incompressible fluid:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

- $\boxed{Re < Re_{tr}\text{: laminar,} \qquad Re > Re_{tr}\text{: turbulent}}$

- System of nonlinear partial differential equations,
  Solution is 'only' possible with numerical methods
- There are methods, by which it is in principle possible to solve the equations:
  $\Rightarrow$ direct numerical simulation (DNS)

The Reynoldsnumber

$$\Longrightarrow \; Re = \overline{U} L / \nu$$

- omroeren van koffie $\qquad Re \sim 10^4$
- luchtstroming om een auto ($50 km/h$) $\qquad Re \sim 3 \cdot 10^6$
- waterstroming in een rivier $\qquad Re \sim 10^7$
- lucht stroming om een vliegtuig $\qquad Re \sim 3 \cdot 10^8$

Questions on precise computations

DNS has two problems

1. Turbulence is deterministic chaos
   $\Rightarrow$ the numerical problem is ill-posed
   Round-off errors, approximation errors etc.
   will result in different flow behavior

- The numerical experiment is like a real experiment,
  $\Rightarrow$ one has to interpret the results in the same way:
  repeat the experiment several times,
  build averages of flow quantities
- All unknowns are known for all time steps in the whole flow domain

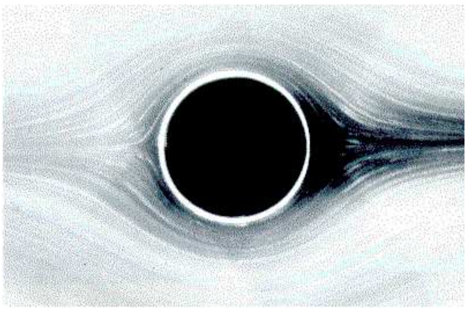
2. In turbulent flows the very small eddies play an important role.
   The computational grid should be such that they are captured.

- Macro structure with length scale $\mathcal{L}$,
  micro structure with the Kolmogorov length scale η
- Number of grid points: $\left(\frac{\mathcal{L}}{\eta}\right)^3$

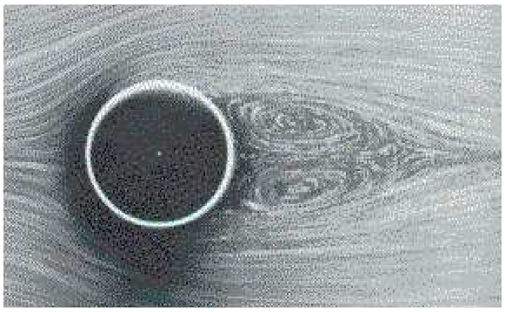
DNS, an example

- channel flow at $Re = \overline{U} \cdot \mathcal{L}/\nu = 10^5$
- smallest length scale $\eta \approx \mathcal{L}/1000$
- resolve the smallest turbulence movements:
  1000 points in each direction: 10^9 points
- computer storage: $> 10 \times 10^9$
- number of computing operations: ~ 500
- number of time steps: ~ 10000
  $\Rightarrow$ Total number of operations 5×10^{15}

- DNS is not yet applicable for practical applications
- With the increasing power of supercomputers, it becomes possible to analyze increasingly larger Reynoldsnumbers.
- Then it is possible to study the basic mechanisms of turbulence.

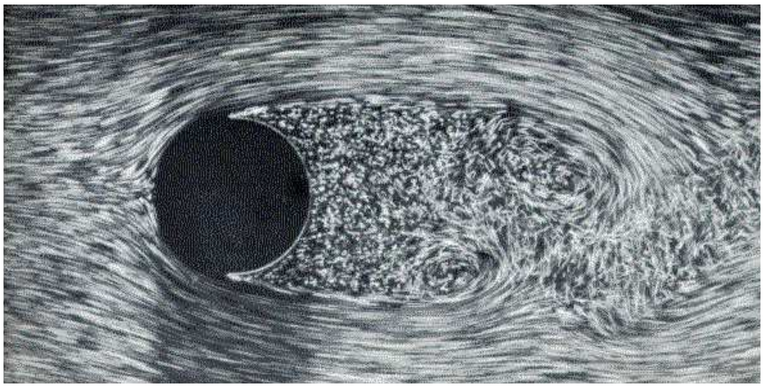


$Re = 0.16$,
Laminar, stationary,
symmetric

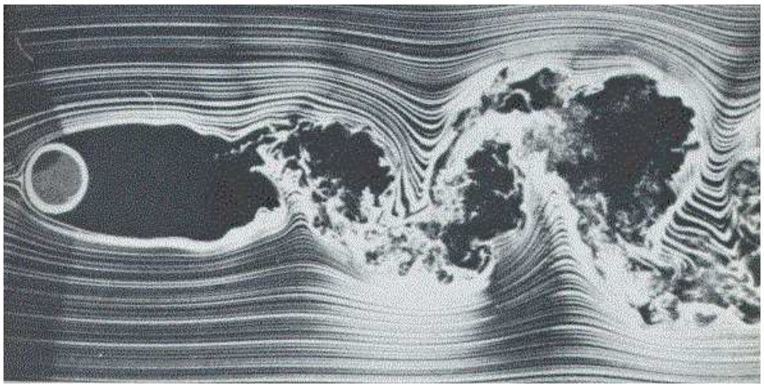


$Re = 26$,
Laminar, stationary,
recirculation

$Re = 140$,
transition, unsteady,
von Kármán vortexstreet



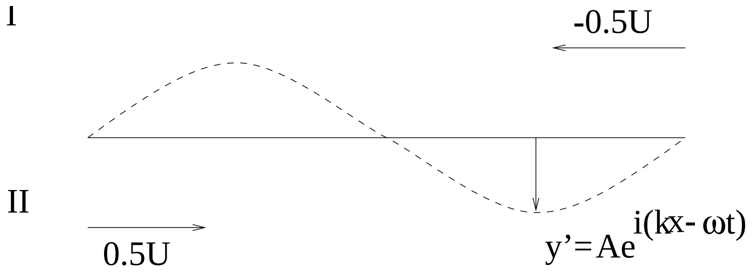
$Re = 2000$,
Turbulent, unsteady



$Re = 10000$,
Turbulent, unsteady

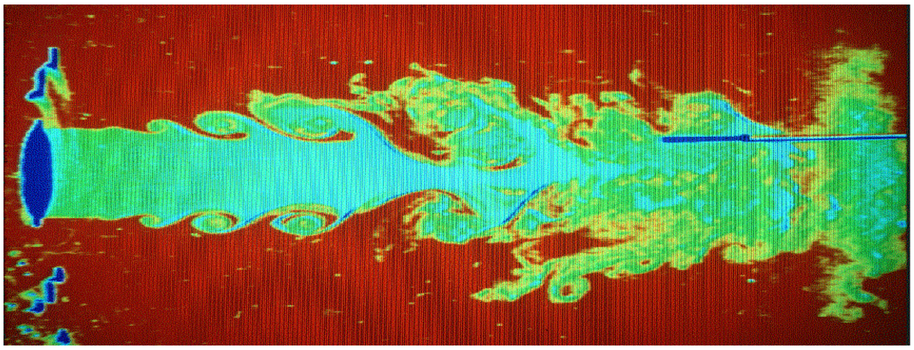
Kelvin-Helmholtz instability

Stability analysis



- $-\infty < x < \infty$, $\quad k = 2\pi/\lambda \in I\!R^+$: wave number
- $\omega = \omega_r + i\omega_i$, $\quad \omega_r = 2\pi/T$
- Assumption: the flow is invisicid and free of rotation (potential flow)
- $\omega_i > 0$: the amplitude grows exponentially $\rightarrow$ unstable flow
- Indeed one obtains: $\omega_1 = \frac{1}{2} i U k$

  $\Rightarrow$ <u>the flow is unstable</u> for initial disturbances



Road to chaos

Bifurcation analysis

- Example: the Lorenz model (1963), a simple model for the Rayleigh-Bénard instability

- A system with 3 time-dependent nonlinear deterministic equations can show a timely irregular behavior

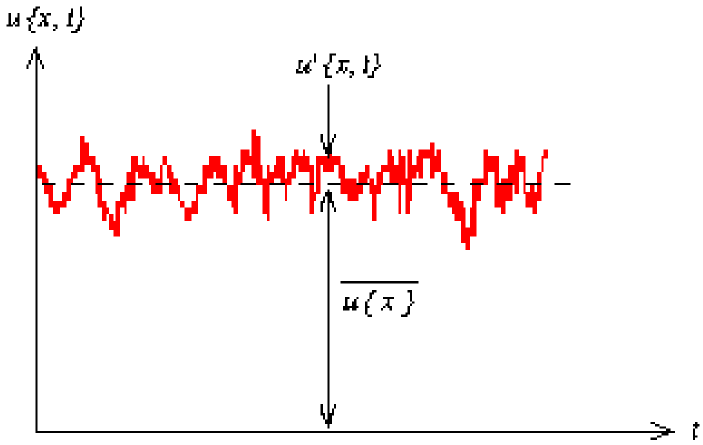
Turbulence mechanism:

- The fastest growing disturbances form the largest eddies. These are unstable, break up in smaller eddies etc. and are replaced continuously.
- The generation and destruction of eddies (cascade process) is a mechanism, that converts the energy of the average flow via viscous dissipation in the smallest eddies into heat

The three models:

- Direct Numerical Simulation
- Averaged Navier-Stokes equations (Reynolds equations)
- "Large Eddy Simulation"

Statistical view



- $u(x,t) \equiv \overline{u} + u' \qquad p = \overline{p} + p'$, etc.
- $\overline{u} = \frac{1}{T}\int_t^{t+T} u(x,t')dt'$ (Stationary process) or
  $\lim_{N\to\infty} \frac{1}{N}\Sigma_{\alpha=1}^{N} u(x,t)^{(\alpha)}$
- $\overline{u'} = 0$
- Averaged equations (Reynolds equations),
  i.e. for x-momentum:

$$\frac{\partial \overline{u}}{\partial t} + \overline{u}\frac{\partial \overline{u}}{\partial x} + \overline{v}\frac{\partial \overline{u}}{\partial y} + \overline{w}\frac{\partial \overline{u}}{\partial z} + \frac{1}{\rho}\frac{\partial \overline{p}}{\partial x} =$$
$$\frac{\partial}{\partial x}\left(\frac{\mu}{\rho}\frac{\partial \overline{u}}{\partial x} - \overline{u'u'}\right) + \frac{\partial}{\partial y}\left(\frac{\mu}{\rho}\frac{\partial \overline{u}}{\partial y} - \overline{u'v'}\right) + \frac{\partial}{\partial z}\left(\frac{\mu}{\rho}\frac{\partial \overline{u}}{\partial z} - \overline{u'w'}\right)$$

(similar for y- and z-momentum)

continuity equation:

$$\frac{\partial \overline{u}}{\partial x} + \frac{\partial \overline{v}}{\partial y} + \frac{\partial \overline{w}}{\partial z} = 0$$

Closure problem

- The closure of the system is only possible by adding empirical information

$$-(\overline{u'u'} + \overline{u'v'} + \overline{u'w'}) = \mu_t \left(\frac{\partial \overline{u}}{\partial x} + \frac{\partial \overline{u}}{\partial y} + \frac{\partial \overline{u}}{\partial z} \right)$$

- Modeling $\mu_t = \rho v_t \ell$ as an eddy viscosity
- For example, by Prandl's mixing length theory (aerodynamics)
- Or by the $k - \varepsilon$ turbulence model (hydrodynamics)

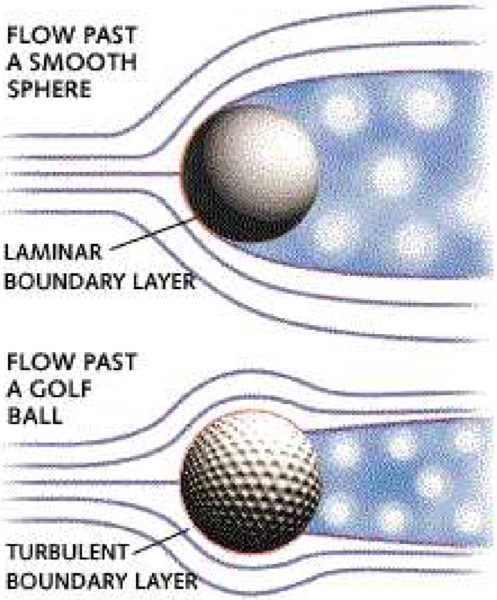
$$v_t \sim \sqrt{k}, \quad \ell \sim k^{3/2}/\varepsilon \quad (k = \frac{1}{2}\overline{u_i' u_i'})$$

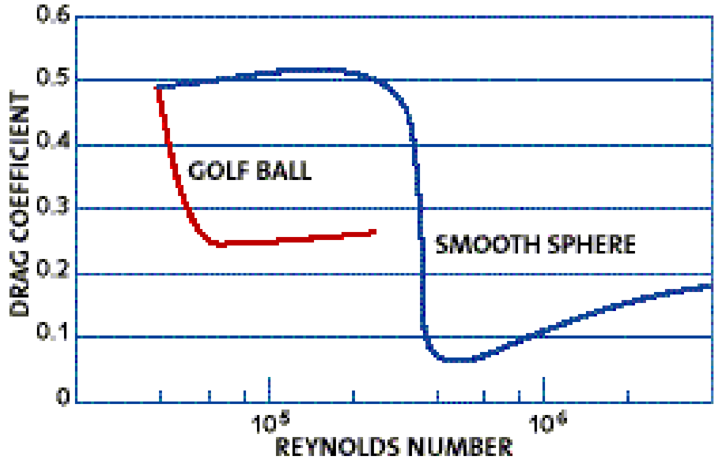
  A model with 2 additional transport equations for k and ε

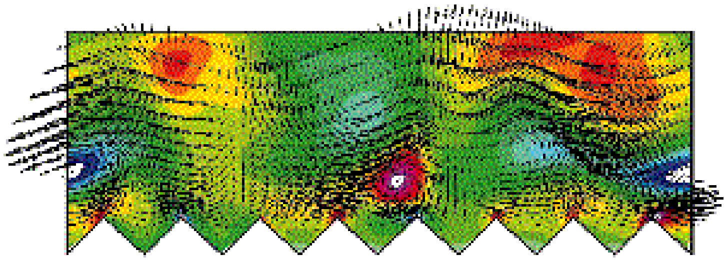
- Many varianten with empirical constants, ad hoc possiblities to change
- Daily used by the engineers
- Compromise between accuracy and technical feasibility of computations

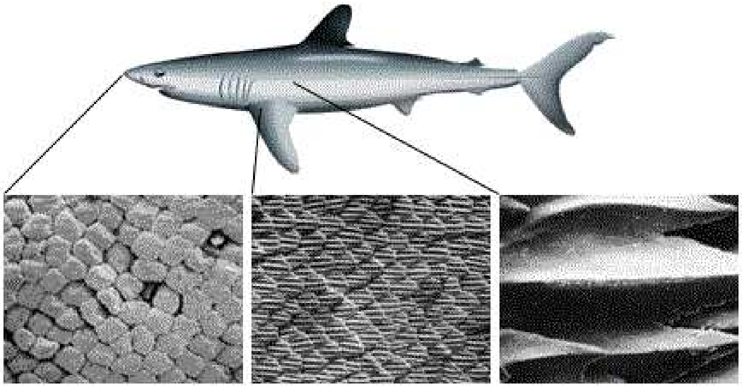
“Large Eddy Simulation” (LES)

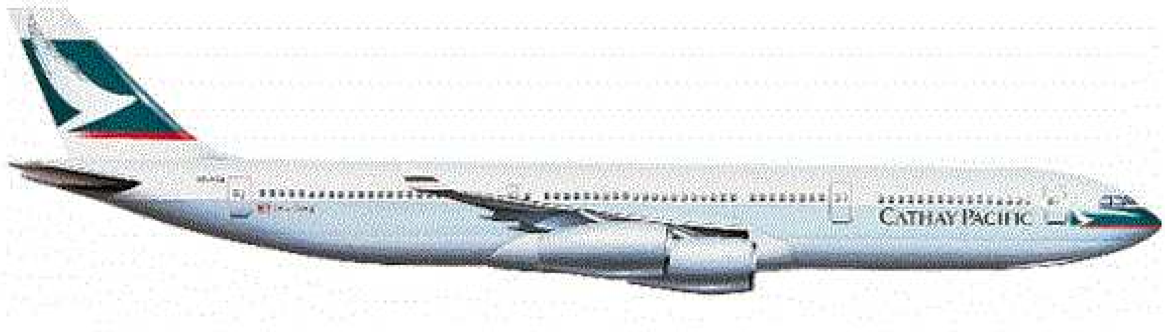
- Different treatment of macro- and micro structure of turbulence
- Macro structure is problem-dependent (computed by DNS)
- 3D, time-dependent computations on a fine mesh, that does not capture the smallest eddies
- One filters these smallest eddies out, and models them, for example, by approaches based on a statistical view on turbulence
- micro structure is universal (?) “isotropic turbulence” and is modelled as in the Reynolds equations with a closure
- Successful in atmospheric convective boundary layer calculations
- LES is a promising approach with initial applications in industry

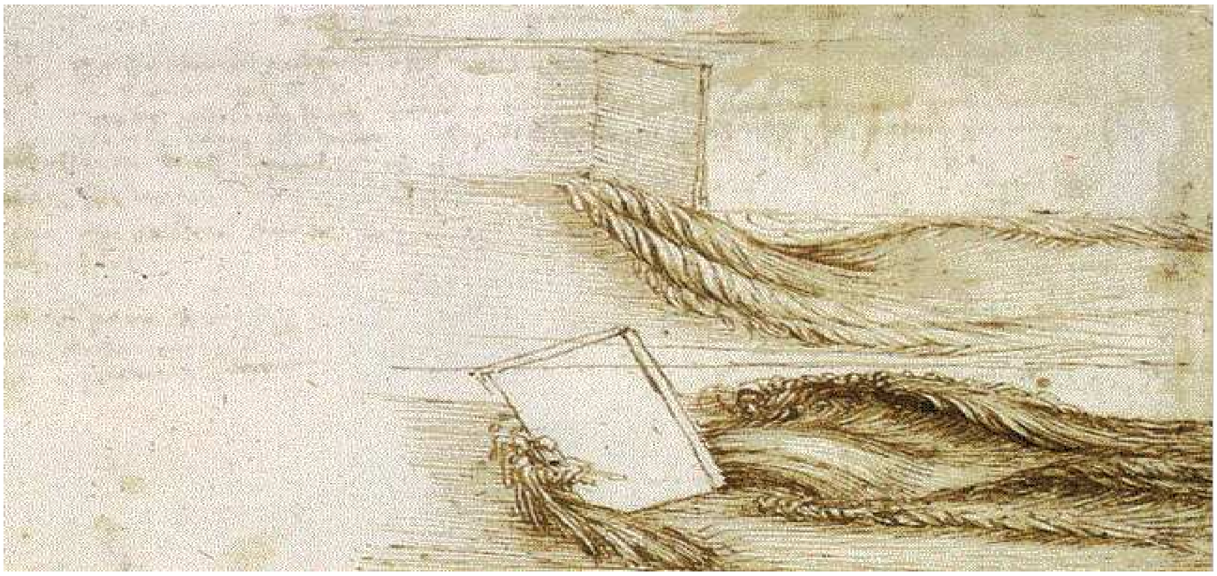

FLOW PAST
A SMOOTH
SPHERE
LAMINAR
BOUNDARY LAYER
FLOW PAST
A GOLF
BALL
TURBULENT
BOUNDARY LAYER


0.6
0.5
0.4
0.3
0.2
0.1
0
DRAG COEFFICIENT
GOLF BALL
SMOOTH SPHERE
10^5
10^6
REYNOLDS NUMBER

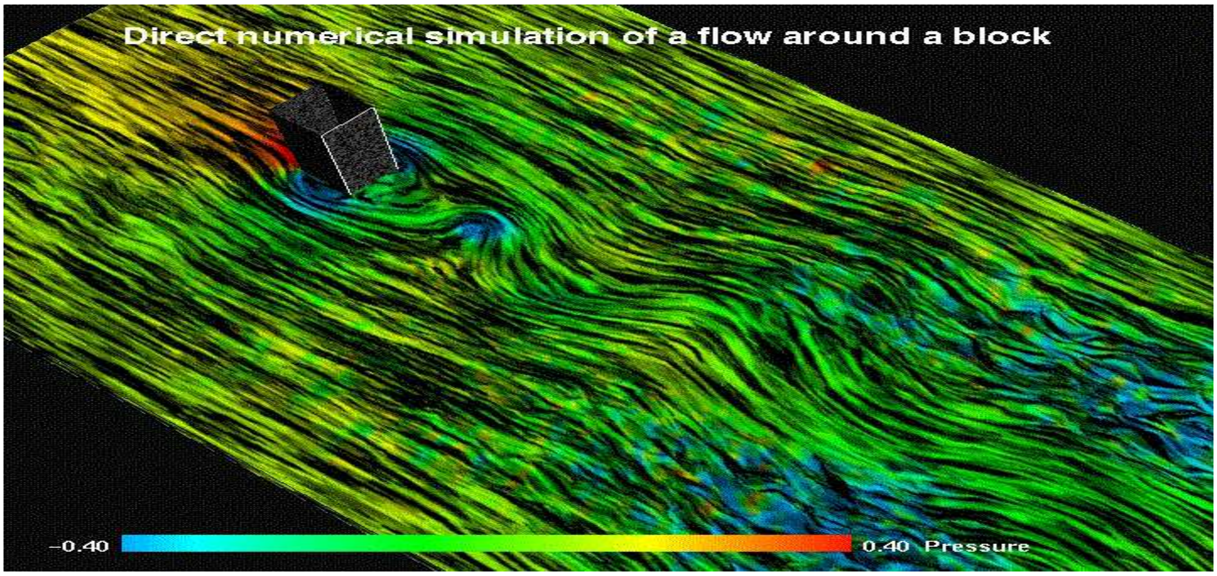





CATHAY PACIFIC



Leonardo



DNS

$Re = 22000$

(numerical simulation, Prof. Veldman, U. Groningen NL)