**Bi-CG**

$x_0$ is given; $r_0 = b - Ax_0$;

$\hat{r}_0$ is an arbitrary vector $(\hat{r}_0, r_0) \neq 0$

possible choice $\hat{r}_0 = r_0$ ;

$\rho_0 = 1$

$\hat{p}_0 = p_0 = 0$

**for** $i = 1, 2, ...$

$\rho_i = (\hat{r}_{i-1}, r_{i-1})$ ; $\beta_i = (\rho_i / \rho_{i-1})$ ;

$p_i = r_{i-1} + \beta_i p_{i-1}$ ;

$\hat{p}_i = \hat{r}_{i-1} + \beta_i \hat{p}_{i-1}$ ;

$v_i = Ap_i$

$\alpha_i = \rho_i / (\hat{p}_i, v_i)$;

$x_i = x_{i-1} + \alpha_i p_i$

$r_i = r_{i-1} - \alpha_i v_i$

$\hat{r}_i = \hat{r}_{i-1} - \alpha_i A^T \hat{p}_i$

**end for**

Bi-CGSTAB method

$x_0$ is an initial guess; $r_0 = b - Ax_0$;

$\bar{r}_0$ is an arbitrary vector, such that $(\bar{r}_0, r_0) \neq 0$, e.g., $\bar{r}_0 = r_0$ ;

$\rho_{-1} = \alpha_{-1} = \omega_{-1} = 1$ ;

$v_{-1} = p_{-1} = 0$ ;

for $i = 0, 1, 2, ...$ do

$\rho_i = (\bar{r}_0, r_i)$ ; $\beta_{i-1} = (\rho_i/\rho_{i-1})(\alpha_{i-1}/\omega_{i-1})$ ;

$p_i = r_i + \beta_{i-1}(p_{i-1} - \omega_{i-1}v_{i-1})$ ;

$\hat{p} = K^{-1}p_i$ ;

$v_i = A\hat{p}$ ;

$\alpha_i = \rho_i/(\bar{r}_0, v_i)$ ;

$s = r_i - \alpha_i v_i$ ;

if $\|s\|$ small enough then

$x_{i+1} = x_i + \alpha_i\hat{p}$ ; quit;

$z = K^{-1}s$ ;

$t = Az$ ;

$\omega_i = (t, s)/(t, t)$ ;

$x_{i+1} = x_i + \alpha_i\hat{p} + \omega_i z$ ;

if $x_{i+1}$ is accurate enough then quit;

$r_{i+1} = s - \omega_i t$ ;

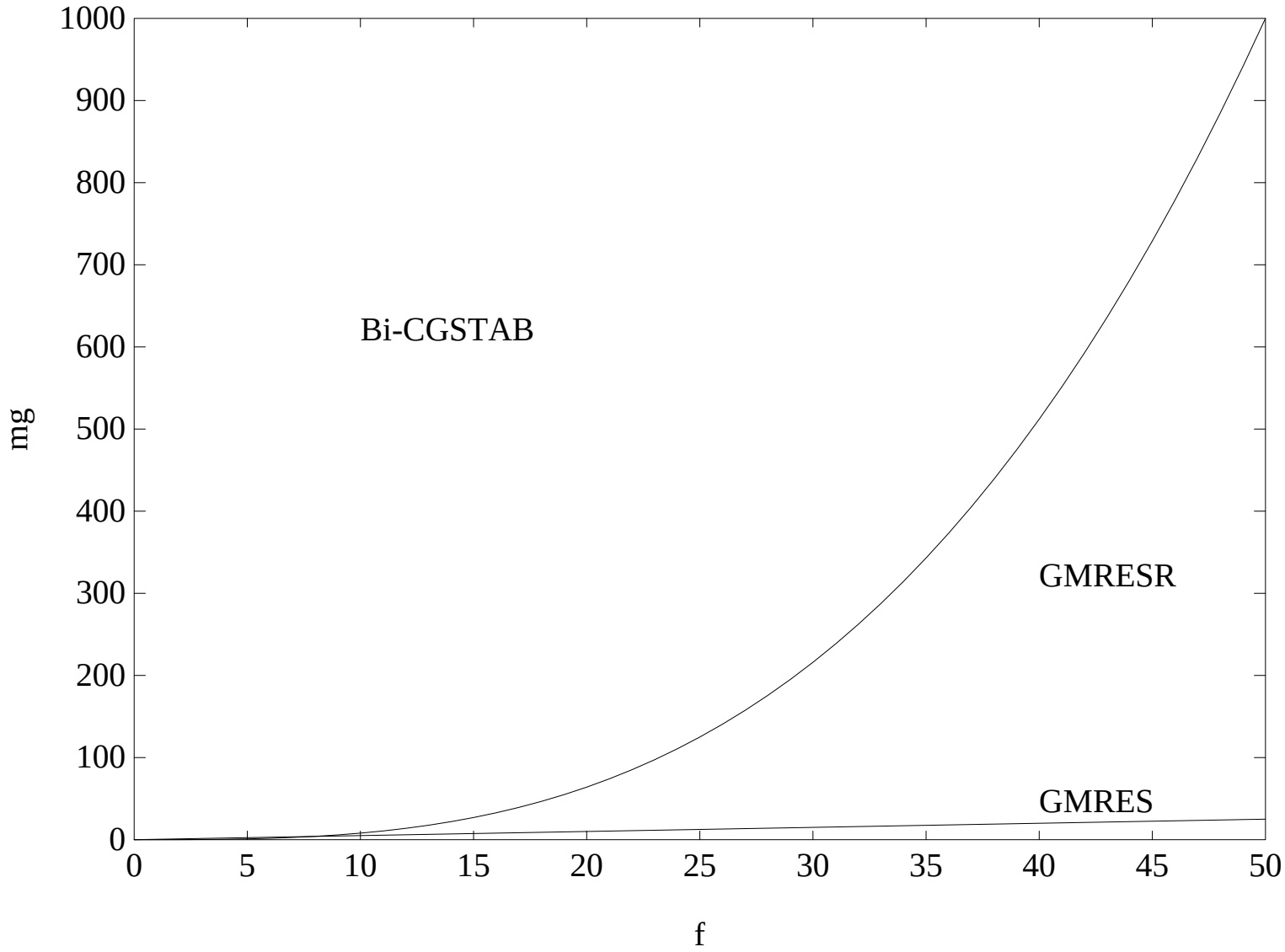
end for

GCR algorithm
choose $x_0$, compute $r_0 = b - Ax_0$

for $i = 1, 2, ...$ do
$s_i = r_{i-1}$ ,
$v_i = As_i$ ,
for $j = 1, ..., i-1$ do
$\alpha = (v_i, v_j)$ ,
$s_i := s_i - \alpha s_j$ , $v_i := v_i - \alpha v_j$ ,
end for
$s_i := s_i / \|v_i\|_2$ , $v_i := v_i / \|v_i\|_2$
$x_i := x_{i-1} + (r_{i-1}, v_i) s_i$ ;
$r_i := r_{i-1} - (r_{i-1}, v_i) v_i$ ;
end for

GMRESR algorithm
choose $x_0$ and $m$, compute $r_0 = b - Ax_0$

for $i = 1, 2, \ldots$ do
  $s_i = P_{m,i-1}(A) r_{i-1}$ ,
  $v_i = A s_i$ ,
  for $j = 1, \ldots, i-1$ do
    $\alpha = (v_i, v_j)$ ,
    $s_i := s_i - \alpha s_j$ , $v_i := v_i - \alpha v_j$ ,
  end for
  $s_i := s_i / \|v_i\|_2$ , $v_i := v_i / \|v_i\|_2$
  $x_i := x_{i-1} + (r_{i-1}, v_i) s_i$ ;
  $r_i := r_{i-1} - (r_{i-1}, v_i) v_i$ ;
end for


1000
900
800
700
600
500
400
300
200
100
0
mg
Bi-CGSTAB
GMRESR
GMRES
0
5
10
15
20
25
30
35
40
45
50
f