Show Me Your Error Bars! Practical Uncertainty Analysis in Numerical Experiments

Advisor: Dr. Neil Budko (n.v.budko@tudelft.nl)

Problem description

A good experimentalist always presents results with error bars. The latter are a good indication of the measurement uncertainty. The experimental curves and their bars are obtained by repeating the measurements under identical (controllable) conditions and recording the mean and the standard deviation of the results over an empirical statistical ensemble of data. Theoreticians and engineers, however, typically assume exact input parameters (boundary conditions, coefficients, source functions, etc.) in their models and produce smooth curves without any error bars. While a lot of effort goes into ensuring the validity and convergence of discretization procedures and numerical algorithms, not much has been achieved in the way of analyzing the uncertainty of numerical models in practice. To be sure, there are many fine theories about the uncertainty/sensitivity of mathematical/numerical models but the error bars are still missing.

Similar to the actual experiments, multiple runs of a numerical simulation with different randomly perturbed inputs can be very expensive. In this Project the student will look into the ways of accelerating the uncertainty analysis in numerical models based on standard PDE's, with the goal of making the computation of error bars easier and thus bringing the numerical experiments on a par with the actual ones.

Approach

1. Literature review (uncertainty, sensitivity, model order reduction)
2. For a typical linear PDE-based numerical model, analyze the corresponding linear algebraic problem and its (iterative) solution algorithm. Describe the changes introduced into the original "exact" problem due to the variations of input parameters. See if the solution of the perturbed problem can be obtained efficiently (e.g. by reduced-order modeling).
3. Show us some error bars!

References

[1] http://en.wikipedia.org/wiki/Experimental_uncertainty_analysis
[2] T. Bui-Thanh et al, Parametric Reduced-Order Models for Probabilistic Analysis of Unsteady Aerodynamic Applications, AIAA JOURNAL, Vol. 46, No. 10, October 2008